Research of Mathematical Economics
No. 1 2008

CONTENTS

A Welfare Analysis of the Predictability of Business Cycles in China
.................................................................Han Xu (5)

Labor Supply: A Perspective On The Basis of Social Preference
.................................................................Liu Kai (21)

A Study on the Difference among the Welfare Costs of Chinese Inflation
.................................................................Zhou Xuan (38)

Infrastructure, Public Services and Optimal Economic Growth
.................................................................Huo Zhen (48)

Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China
.................................................................Huo Zhen (62)

Labor Market Distortion and Price Adjustment: Static Analysis on Chinese Economy within a General Equilibrium Framework
.................................................................Qiu Zhasheng(77)

Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium
.................................................................Zhang Lei (97)
目录

中国经济波动可预测性的福利分析
........................................................................................................... 韩 旭 (5)

劳动供给：一个基于社会偏好的视角
........................................................................................................... 刘 凯(21)

通货膨胀福利成本的差异性研究
........................................................................................................... 周 璐(38)

基础设施，公共服务与最优经济增长
........................................................................................................... 霍 震(48)

中国财政分权，公共支出构成与经济增长
........................................................................................................... 霍 震(62)

一般均衡下的劳动力市场扭曲和价格调整：基于中国经济的静态分析
........................................................................................................... 邱哲圣(77)

最优资本税收和投资信贷的一般均衡分析
........................................................................................................... 张 磊(97)
Research of Mathematical Economics

Organizers:
Economics and Mathematics Double Bachelor Program
Renmin University of China

Academic Adviser:
Yang Ruilong  Liu Fengliang  Liu Yuanchun  Zhang Yu  Liu Rui Yu Tongshen
Guo Jie  Wang Jibin

Editorial committee:
Li Tianyou  Zhang Yanhong  Yu Ze  Wang Xianghong  Cheng Hua

Instruct Teacher:
Chen Yanbin

Editorial Director:
Ai Yi  Qiu Zhiheng  Zhang Lei  Liu Jingrong  Shen Jing  Guo Yumei  Lu Hong

《数理经济学研究》编辑机构

主办单位： 中国人民大学经济学—数学（双学位）实验班
学术顾问： 杨瑞龙  刘风良  刘元春  张 宇  刘 瑞  丁同申  郭 杰  王晋斌
编 委： 李天有  章艳红  于 泽  王湘红  程 华
指导老师： 陈彦斌
编辑部： 艾 宜（主任）  邱哲圣  张 磊  刘婧蓉  申 婷  郭豫媚  陆 洪
A Welfare Analysis of the Predictability of Business Cycles in China

Han Xu
(School of Economics, Renmin University of China)

Abstract: Lucas (1987) applied a groundbreaking model to measure the welfare cost of business cycles and reduced growth. The disturbance of consumption in his model was assumed to be independent and identically distributed, which is rarely consistent with the actual data. To solve this problem, this paper assumes the disturbance is a first-order autoregressive process, proves the existence of the welfare cost of business cycles and reduced growth, presents the formula and re-estimates these costs based on the consumption data in China. There are two main findings in this paper: the first is the positive correlation between the welfare cost of business cycles and the absolute value of AR (1) coefficient and the second is the lower estimated costs compared to the case of independent and identical distributed disturbance.

Keywords: Economic Fluctuations, Economic Growth, Predictability, Welfare Cost

JEL classification: E32, O40, I31

1. Introduction

When designing macroeconomic policies, governments always face a trade-off between high economic growth and economic stability. Although both high growth and a stable economy can improve consumers’ welfare level, they cannot coexist in reality sometimes: high growth rate may be combined with increased economic fluctuation and low fluctuation is sometimes associated with low economic growth. To weigh the gains and losses associated with these two choices, Lucas (1987, 2003) proposed a model and derived some interesting findings. If the growth rate was reduced from 3% to 2%, the consumption per year should approximately increase by 20% to compensate for the welfare losses of a representative household. However, the welfare losses of business cycles could be compensated for by an increase of 0.008% in consumption, which meant that there existed a great disparity between the welfare cost of business cycles and that of reduced

---

1 This paper was part of the joint paper with Professor Chen Yanbin.
A Welfare Analysis of the Predictability of Business Cycles in China

growth. Therefore, Lucas concluded that governments should be more concerned with economic growth and less concerned with economic fluctuations due to the extremely low gains from stabilization policies.

Lucas's conclusion was followed by significant research on this topic. Some researchers derived higher estimated values for the welfare cost of business cycles. For example, Pallage and Robe (2001) calculated the welfare cost of business cycles in 11 African countries including South Africa and found that the average welfare cost in these counties was about 25 times higher than that in the United States. Epaulard, Pommeret (2003), Dolmas (1998), Ayse Imrohoroglu, Selahattin Imrohoroglu (1997), Van Wincoop (1994) also drew similar conclusions. However, some researchers, including, Otrok (2001), Alvarez, Jermann (2004), estimated the welfare cost of business cycles to approximate Lucas's original calculation.

All results mentioned above are based on data from foreign countries. It is dubious to directly apply these results in China, which has an economy that is currently in a transitional stage, from a centrally-planned economy to a market economy. Therefore, Chen and Zhou (2004) established a model based on habit formation to measure the welfare cost of business cycles in China; they found this cost to be 22 times higher in China than that in the United States. Also, when calculating the welfare cost of business cycles and reduced growth in America, Lucas applied different forms of utility functions that caused a large gap between these two kinds of welfare costs (Chen, 2005). After taking account of the high growth rate and the greater fluctuation of the Chinese economy, Chen found that these two kinds of welfare costs were quite similar to each other, which meant that the Chinese government should focus on both maintaining a high growth rate as well as the stability of the economy.

However, disturbance of consumption stream is always assumed to be independent and identically distributed in the existing literature and the probability of serial correlation is omitted, which may lead to inaccurate estimates of the value of the welfare cost of business cycles and reduced growth. In fact, autocorrelation often exists in time series data on consumption stream. Abel (1990) estimated that the growth rate of consumption in the United States was a first-order autoregressive process, and the AR(1) coefficient is -0.14. Therefore, it is necessary to incorporate autocorrelation into Lucas's model. Since autocorrelation may cause the model to become very complicated, while the AR(1) process is relatively simple and retains the main features of autocorrelation, we apply the AR(1) process instead of independent and identical distribution
Research of Mathematical Economics No. 1 2008

(i.i.d.) in our model. Note that the i.i.d. assumption is a special case of AR(1) process when the AR(1) coefficient is zero, so our model extends Lucas’s model.

The structure of this paper is organized as follows. In part 2, the i.i.d. assumption is substituted by the AR(1) process, the expressions of the welfare cost of business cycles and reduced growth will be re-deduced according to the new assumptions and we will discuss the new model’s differences with Lucas’s model. In part 3, parameters, such as the growth rate of consumption in China and the AR(1) coefficient, are estimated based on the modified model. Then these values are applied to calculate the welfare cost of business cycles and reduced growth, which is given in part 4. Finally, we draw conclusions in part 5.

2. The Model

We concisely review Lucas’s model. It is assumed that time is discrete, population is fixed and the utility function has the CRRA form. Therefore, the expected life-time utility of a representative household is:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right\}$$

Where $C_t$ is his consumption in period $t$, $\beta$ is the subjective time discount factor, and $\gamma$ is the relative risk-aversion coefficient. Lucas assumed that the stochastic consumption stream is $C_t = A \phi^{\mu_t^*} e^{-0.5\sigma^2 t^*} e^n$, where $e_t$ was a normally distributed random variable with mean 0 and variance $\sigma^2$. Lucas used a compensation parameter $\lambda$ to measure the welfare gains from eliminating consumption disturbance. Since a risk-averse consumer would prefer a deterministic consumption path to a risky path with the same mean, Lucas qualified this utility difference by multiplying the risky path by a constant factor $1+\lambda$ in all dates and states, choosing $\lambda$ so that the household was indifferent between the deterministic stream and the compensated, risky consumption stream, i.e.

$$E \sum_{t=n}^{\infty} \beta^t \left[ 1 + \lambda \right] \left( A \phi^{\mu_t^*} e^{-0.5\sigma^2 t^*} e^n \right)^{-\gamma} = \sum_{t=n}^{\infty} \beta^t \left[ \frac{E \left( A \phi^{\mu_t^*} e^{-0.5\sigma^2 t^*} e^n \right)}{1-\gamma} \right]^{-\gamma}$$

Given formula (2), Lucas proved that the welfare cost of business cycles is about $0.5\sigma \gamma e^2$.

Similarly, Lucas also used a compensation coefficient $\delta$ to calculate the welfare losses when
A Welfare Analysis of the Predictability of Business Cycles in China

the growth rate was decreased from \( \mu \) to \( \mu_t \). However, the utility function was in log form, which was a special case of the CRRA function when the relative risk-aversion coefficient was 1. If we use the CRRA preference to re-solve the problem, then:

\[
E \sum_{t=0}^{\infty} \beta^t \left[ (1 + \delta)(Ae^{\mu_t}e^{0.5\sigma^2}e^{\varepsilon_t}) \right]^{1-\gamma} = E \sum_{t=0}^{\infty} \beta^t \left( Ae^{\mu_t}e^{0.5\sigma^2}e^{\varepsilon_t} \right)^{1-\gamma} \tag{3}
\]

Chen (2005) proved that based on the CRRA preference, the expression of welfare cost of reduced growth is

\[
\delta = \left[ \frac{1 - \beta (1 + \mu)}{1 - \beta (1 + \mu)} \right]^{1-\gamma} - 1.
\]

In this paper, we follow the idea of Lucas’s compensation coefficient and the other main assumptions in his model. We differ by studying how autocorrelation of the disturbance affects the welfare cost, when \( \varepsilon_t \) in consumption stream \( C_t = Ae^{\mu_t}e^{0.5\sigma^2}e^{\varepsilon_t} \) is assumed to be an AR (1) process, i.e.

\[
\varepsilon_t = \rho \varepsilon_{t-1} + \nu_t, \quad \nu_t \sim i.i.d. N(0, \sigma^2) \tag{4}
\]

Under the i.i.d. assumption, Lucas applied unconditional expectation to calculate the welfare cost. In fact, this method can be treated as a special case of conditional expectation and it reduces to the unconditional case because the \( \varepsilon_t \) is independent and identically distributed. When \( \varepsilon_t \) is assumed to be an AR(1) process, \( \varepsilon_t \) is no longer independent, so we need to use conditional expectation to solve the problem. Note that \( \varepsilon_{t-1} \) includes all information in the past, so \( \varepsilon_{t-1} \) is enough for us to solve the conditional expectation of \( \varepsilon_t \) from period 0 to the infinity. Therefore, the equation to solve the parameter \( \lambda \) by conditional expectation can be shown by the following formula:

\[
E \lambda = \sum_{t=0}^{\infty} \beta^t \left[ (1 + \lambda)(Ae^{\mu_t}e^{0.5\sigma^2}e^{\varepsilon_t}) \right]^{1-\gamma} = \sum_{t=0}^{\infty} \beta^t \left( E_0(Ae^{\mu_t}e^{0.5\sigma^2}e^{\varepsilon_t}) \right)^{1-\gamma} \tag{5}
\]

Given formulas (4) and (5), we can re-calculate the welfare cost of business cycles and the result is given by proposition 1.

**Proposition 1:** Suppose that the utility function has the CRRA form and the disturbance of consumption is an AR(1) process with coefficient \( \rho \), then the welfare cost of business cycles is:

\[
\lambda = \left\{ \sum_0^{\infty} X_t / \sum_0^{\infty} Y_t \right\}^{1/\gamma} - 1 \tag{6}
\]

Where:
Research of Mathematical Economics No. 1 2008

\[
X_t = \beta e^{\rho_{t-1} e_t} e^{\rho_{t-1}(1-\gamma) e_t (1-p^{2s}) \sigma^2 (1-\gamma) / (2(1-p^2))}
\]

\[
Y_t = \beta e^{\rho_{t-1} e_t} e^{\rho_{t-1}(1-\gamma) e_t (1-p^{2s}) \sigma^2 (1-\gamma) / (2(1-p^2))}
\]

Proof: given formula (4), we can derive \( e_t = \rho^{t+1} e_{-1} + \rho^t \nu_0 + \rho^{t-1} \nu_1 + \ldots + \rho \nu_{t-1} + \nu_t \), then it is easy to know that the conditional expectation and variance of \( e_t \) are \( \rho^{t+1} e_{-1} \) and \((\rho^{2t} + \ldots + \rho^2 + 1)\sigma^2 \) respectively. Hence we derive the conditional expectation of \( e^\gamma \) as:

\[
E_\gamma(e^\gamma) = e^{\rho^{t+1} e_{-1} + 0.5(\rho^{2t} + \ldots + \rho^2 + 1)\sigma^2} = e^{\rho^{t+1} e_{-1} + (1-p^{2s}) \sigma^2 / (2(1-p^2))}
\]

Substitute this result into the expression of \( C_\gamma \) and then we can solve \((E_\gamma C_\gamma)^{1-\gamma}\) and \(E_\gamma(C_\gamma^{1-\gamma})\) as follows:

\[
[E_\gamma(C_\gamma)]^{1-\gamma} = [E_\gamma(Ae^{\mu e_t} e^{-0.5\sigma^2 e_t})]^{1-\gamma}
\]

\[
= [\frac{\begin{array}{c}
\sum_{t=0}^{\infty} B'(e^{\mu e_t}) (e^{\rho^{t+1} e_{-1} + (1-p^{2s}) \sigma^2 / (2(1-p^2))})^{1-\gamma}
\end{array}}{\sum_{t=0}^{\infty} B'(e^{\mu e_t})} - 1]
\]

According to Lucas's view, the compensation parameter \( \lambda \) depends on the variance of the disturbance and the relative risk-aversion coefficient. However, when generalizing the model by the AR(1) assumption, we found that the factors that can affect the welfare cost become more complicated and making it difficult to obtain an explicit solution. Also, to guarantee the existence and uniqueness of \( \lambda \), the sums of the functional series, \( \sum_{t=0}^{\infty} X_t \) and \( \sum_{t=0}^{\infty} Y_t \), must be convergent. [Proof ends]

**Proposition 2:** when the absolute value of \( \rho \) is smaller than 1, \( \sum_{t=0}^{\infty} X_t \) and \( \sum_{t=0}^{\infty} Y_t \) given in formula (6) will be convergent if and only if \( \beta e^{\mu(1-\gamma)} < 1 \).

Proof: “ \( \Rightarrow \)”, it is trivial to test that both series \( \{ e^{\rho^{t+1} e_{-1} + (1-p^{2s}) \sigma^2 / (2(1-p^2))} \} \) and
A Welfare Analysis of the Predictability of Business Cycles in China

\[ \left\{ e^{\mu t - \rho t^2/2} \right\} \text{ have suprema because the absolute value of } \rho \text{ is smaller than 1. Denote them as } S_1 \text{ and } S_2 \text{ respectively and then:} \]

\[ 0 < \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2} < S_1 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \]

\[ 0 < \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2} < S_2 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \]

Since \( \beta e^{\mu t - \rho t^2/2} < 1 \), the sums of series \( S_1 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \) and \( S_2 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \) are both convergent. Therefore, \( \sum_{t=0}^{\infty} X_t \) and \( \sum_{t=0}^{\infty} Y_t \) are also convergent.

\[ \text{“} \Leftarrow \text{”}, \text{ Since if } \sum_{t=0}^{\infty} X_t \text{ and } \sum_{t=0}^{\infty} Y_t \text{ are convergent respectively, } \beta e^{\mu t - \rho t^2/2} < 1 \Rightarrow \text{if } \beta e^{\mu t - \rho t^2/2} \geq 1 \text{, then at least one of } \sum_{t=0}^{\infty} X_t \text{ and } \sum_{t=0}^{\infty} Y_t \text{ is divergent, we can prove the latter one. It is also trivial to test that both series } \left\{ e^{\mu t - \rho t^2/2} \right\} \text{ and } \left\{ e^{\mu t - \rho t^2/2} \right\} \text{ have infima because the absolute value of } \rho \text{ is smaller than 1. Denote them as } J_1 \text{ and } J_2 \text{ respectively and then:} \]

\[ 0 < J_1 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2} < S_1 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \]

\[ 0 < J_2 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2} < S_2 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \]

Since \( \beta e^{\mu t - \rho t^2/2} \geq 1 \), the sums of series \( J_1 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \) and \( J_2 \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] \) are both divergent. Therefore, \( \sum_{t=0}^{\infty} X_t \) and \( \sum_{t=0}^{\infty} Y_t \) are also divergent. [Proof ends]

Now we will re-calculate the welfare cost of reduced growth under the AR(1) assumption. Using conditional expectation to rewrite formula (3) gives:

\[ E_1 \sum_{t=0}^{\infty} \beta^{t-1}\left[ (1+\delta)(Ae^{\mu t - 0.5\sigma^2} e^{\mu t}) \right]^{1-\gamma} = E_1 \sum_{t=0}^{\infty} \beta^{t-1}\left( Ae^{\mu t - 0.5\sigma^2} e^{\mu t} \right)^{1-\gamma} \]

(7)

Given formula (4) and the proof process of proposition 1, the expression of \( \delta \) can be solved and is shown below:

**Proposition 3**: Suppose that the utility function has the CRRA form and the disturbance of consumption is an AR(1) process with coefficient \( \rho \), then the welfare cost of a decrease in growth rate from \( \mu \) to \( \mu_t \) is:

\[ \delta = \left\{ \frac{\sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2} \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2}}{\sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2} \sum_{t=0}^{\infty} \left[ \beta e^{\mu t - \rho t^2/2} \right] e^{\mu t - \rho t^2/2}} \right\}^{1-\gamma} - 1 \]

(8)
Proof: It has been proven that  
\[ E_n[(C_t)^{1-\gamma}] = (Ae^{\mu t}e^{-0.5\sigma^2})^{1-\gamma} e^{\mu \epsilon_{n,1}} e^{(1-\gamma)(1-\gamma/2)} \]
Substituting this term into formula (7), canceling and collecting terms will give the result shown in proposition 3. [Proof ends]

Note the series given in formula (8) are quite similar with those given in proposition 1, so using the results derived in proposition 2, we can deduce that the series in formula (8) will be convergent if and only if  \( \beta e^{\mu(1-\gamma)} < 1 \) and \( \beta e^{\rho(1-\gamma)} < 1 \).

The welfare cost of reduced growth shown in formula (8) is quite different from that derived under the i.i.d. assumption. Factors that can affect welfare cost of reduced growth are \( \beta, \mu, \mu_t \) and \( \gamma \) under the i.i.d. assumption. However, after the model is modified by the AR(1) assumption, more factors, such as the AR(1) coefficient, the standard error of \( \omega_t \) and \( \epsilon_{-1} \), can also impact the welfare cost of reduced growth.

On the other hand, formula (8) also retains an important feature that is derived from the i.i.d. case, i.e., \( \delta \) decreases with parameter \( \mu_t \). The proposition and proof are listed below:

**Proposition 4:** The welfare cost \( \delta \) given by formula (8) decreases with growth rate \( \mu_t \) (\( \mu_t \leq \mu \) and \( \gamma \neq 1 \)).

Proof:

If \( \gamma > 1 \), \( \beta e^{\mu(1-\gamma)} \) decreases with \( \mu_t \), and every term of the series in the denominator of formula (8) decreases while the numerator remains unchanged. Therefore, the ratio of the numerator to the denominator increases. Note that \( 1/(1-\gamma) < 0 \), so \( \delta \) decreases with \( \mu_t \).

If \( \gamma < 1 \), \( \beta e^{\mu(1-\gamma)} \) increases with \( \mu_t \), and every term of the series in the denominator of formula (8) increases while the numerator remains unchanged. Therefore, ratio of the numerator to the denominator decreases. Note that \( 1/(1-\gamma) > 0 \), so \( \delta \) decreases with \( \mu_t \). [Proof ends]

Proposition 4 implies that the more the growth rate is reduced, the higher the welfare cost becomes, which is consistent with intuition as well as the conclusion drawn from the i.i.d. assumption.

The calculations of propositions 1 and 3 illustrate that using conditional expectation to solve this problem results in functions of \( \epsilon_{-1} \). Hence the value of \( \epsilon_{-1} \) is needed when we try to apply a numerical simulation to calculate the welfare cost. This is not a simple task because \( \epsilon_{-1} \) is unknown. Therefore, we can try another way to re-solve the problems by using unconditional
A Welfare Analysis of the Predictability of Business Cycles in China

expectation. The results are listed below in proposition 5.

Proposition 5: Suppose the utility function has the CRRA form, the disturbance of consumption is an AR(1) process with coefficient \( \rho \) and unconditional expectation is applied instead of conditional expectation, then the welfare cost of business cycles and reduced growth is:

\[
\lambda = e^{0.5\gamma^2/(1-\rho^2)} - 1 \approx 0.5\gamma^2/(1-\rho^2)
\]

(9)

\[
\delta = \left[ \frac{1 - \beta e^{\mu/(1-\gamma)}}{1 - \beta e^{\mu/(1-\gamma)}} \right]^{1/\gamma} - 1 \approx \left[ \frac{1 - \beta (1 + \mu)^{1-\gamma}}{1 - \beta (1 + \mu)^{1-\gamma}} \right]^{1/\gamma} - 1
\]

(10)

In addition, if other factors remain constant, the welfare cost increases with the absolute value of the first-order autoregressive coefficient \( \rho \), i.e. the stronger the autocorrelation of the disturbance, the higher the welfare cost of business cycles. On the other hand, the welfare cost of the reduced growth rate is not affected by the autocorrelation of the disturbance of consumption.

Proof: Note that unconditional expectation has the feature that \( E(.) = E[E(.)'] \), so we only need to use unconditional expectation on both sides of formula (5) and (7). Given formula (4), the unconditional expectation of \( e_{-1} \) is \( \sigma^2/(1-\rho^2) \), so \( E(e_{-1}) = e^{0.5\sigma^2/(1-\rho^2)} \). Also, the proof of proposition 1 has given:

\[
[E_{-1}(C_i)]^{1-\gamma} = (AE^{\mu\epsilon}e^{-0.5\sigma^2})^{1-\gamma} e^{\rho_{i}e_{-1}(1-\gamma)/(1-\rho^2)\sigma^2/2(1-\rho^2)}
\]

\[
E_{-1}(C_i^{1-\gamma}) = (AE^{\mu\epsilon}e^{-0.5\sigma^2})^{1-\gamma} e^{\rho_{i}e_{-1}(1-\gamma)/(1-\rho^2)\sigma^2/2(1-\rho^2)}
\]

So the following equations hold automatically:

\[
[E(E_{-1}(C_i))]^{1-\gamma} = (AE^{\mu\epsilon}e^{-0.5\sigma^2})^{1-\gamma} e^{\rho_{i}e_{-1}(1-\gamma)/(1-\rho^2)\sigma^2/2(1-\rho^2)}
\]

\[
= (AE^{\mu\epsilon}e^{-0.5\sigma^2})^{1-\gamma} e^{\rho_{i}e_{-1}(1-\gamma)/(1-\rho^2)\sigma^2/2(1-\rho^2)}
\]

\[
E(E_{-1}(C_i^{1-\gamma})) = (AE^{\mu\epsilon}e^{-0.5\sigma^2})^{1-\gamma} e^{\rho_{i}e_{-1}(1-\gamma)/(1-\rho^2)\sigma^2/2(1-\rho^2)}
\]

\[
= (AE^{\mu\epsilon}e^{-0.5\sigma^2})^{1-\gamma} e^{\rho_{i}e_{-1}(1-\gamma)/(1-\rho^2)\sigma^2/2(1-\rho^2)}
\]

Substituting the above expression into formulas (2) and (3) respectively gives the results shown by formulas (9) and (10). Given formulas (9) and (10), it is trivial to prove that \( \lambda \) increases with the absolute value of \( \rho \) and \( \rho \) has no effects on \( \delta \). [Proof ends]

Compared to the outcome given by using conditional expectation, unconditional expectation gives a more simplified expression of the welfare cost of business cycles and reduced growth. \( \lambda \) is affected by three factors, including relative risk-aversion coefficient \( \gamma \), first-order autoregressive coefficient \( \rho \), and the standard error \( \sigma \) of the white noise series \( \nu \). Therefore, when other factors
are fixed, the welfare cost of business cycles increases with the autocorrelation of economic fluctuations. Also, formula (10) reveals that if unconditional expectation is applied to solve the problem, the welfare cost of reduced growth will reduce to the one derived from the i.i.d. assumption and will no longer be affected by the autocorrelation of disturbance of consumption.

One point that should be emphasized here is that although $\sigma^2(1-\rho^2)$ can be regarded as the variance of $\epsilon$, this does not mean that the modified model in this paper will reduce to the Lucas case. This is because incorporating autocorrelation into Lucas’s model will not only have some impact on the theoretical framework, but also significantly change the econometric estimate. Based on the AR(1) assumption, the estimated value of parameters will be different from the ones estimated in the i.i.d. case. Therefore, the estimated welfare cost in this paper will be different and more precise than the results drawn from the i.i.d. assumption.

3. Estimation of Parameters

In this part, we will estimate the value of parameters $\mu, \rho$ and $\sigma$ in the stochastic consumption stream based on the data from China, and then substitute these numbers into formulas (9) and (10) to calculate the welfare cost of business cycles and reduced growth.

Taking logs on both sides of the expression of stochastic consumption stream gives:

$$\log C_t = (\log A - 0.5\sigma^2) + \mu t + \epsilon_t, \quad \epsilon_t = \rho \epsilon_{t-1} + \nu_t, \tag{11}$$

Since the error terms have the problem of autocorrelation, OLS cannot give an efficient estimation of parameter $\mu$; instead, we can first estimate $\rho$. Rewriting formula (11) with a 1-period lag, we derive:

$$\log C_{t-1} = (\log A - 0.5\sigma^2) + \mu(t-1) + \epsilon_{t-1} \tag{12}$$

Formula (11) minus $\rho$ times formula (12) and collecting terms gives:

$$\log C_t - \rho \log C_{t-1} = (\log A - 0.5\sigma^2)(1-\rho) + \rho \mu + (1-\rho)\mu t + \nu_t \tag{13}$$

It is trivial to prove that formula (13) satisfies the basic assumptions of OLS, so OLS can give an unbiased and efficient estimate of $(1-\rho)\mu$. Then the estimated growth rate of consumption can be calculated by knowing $\hat{\rho}$. Since formula (11) gives the consistent estimated value of $\mu$, we
A Welfare Analysis of the Predictability of Business Cycles in China

can estimate the first-order autoregressive coefficient $\rho$ by doing a regression on $\varepsilon_i$ (Greene, 2003).

Chen and Zhou (2005) used China Statistical Yearbook and established a data set of real consumption per capita in China from 1985 to 2003. They estimated the equation $\log C_i = (\log A - 0.5\sigma^2) + \mu + \varepsilon_i$, where $\varepsilon_i$ was a white noise series. The estimated value of $\sigma$ is 0.055172. We still use the same data in this paper, estimate the formula (11) and do a first-order auto-regression on series $\varepsilon_i$. The result is shown below:

$$\varepsilon_i = 0.693485\varepsilon_{i-1} + \nu_i \quad (0.0003)$$

Substituting $\rho$ into formula (13) and estimating by OLS gives:

$$\log C_i - 0.693485\log C_{i-1} = 1.715583 + 0.020468t \quad (0.0000) \quad (0.0000)$$

$$\text{Adjusted } R^2=0.912394 \quad F=178.0505 \quad \sigma=0.033764$$

Since the estimated values of $\rho$ and $(1-\rho)\mu$ are known, $\mu$ can be calculated and its estimated value is 0.066777, which differs slightly from Chen and Zhou’s estimate. Under the assumption that the error terms are independent and identically distributed, the estimated values of the parameters in the models are: $\sigma=0.055172$ and $\mu=0.060731$. After considering the autocorrelation, the growth rate of consumption $\mu$ is slightly higher while the estimated standard error of $\varepsilon_i$, $\sigma/\sqrt{(1-\rho^2)}$, is equal to 0.0468636, which is smaller than $\sigma=0.055172$, derived in the i.i.d. case. Also, since calculating precise value of $\gamma$ in China is quite difficult and different methods may give different estimates, we continue to use the method given by Lucas (1987) and Ayse Imrohoroglu and Selahattin Imrohoroglu (1997); i.e. choosing several typical values: 1, 5, 10 and 20. Given these parameters values, the welfare cost of business cycles and reduced growth can be calculated by numerical simulation.

4. Numerical Simulation

Table 1 gives the estimated welfare cost of business cycles in China under the i.i.d. and AR(1)

---

2 Series $\nu_i$ has passed the white noise test.
Research of Mathematical Economics No. 1 2008

assumptions respectively. The welfare cost of business cycles increases with the relative risk-aversion coefficient under both assumptions. Moreover, if the standard error of $\varepsilon_n$, $\sigma_n$, is equal to the standard error of $\nu_n$, $\sigma$, then the welfare cost is relative smaller in the i.i.d. case, because the existence of autocorrelation will cause the standard error of $\varepsilon_n$ to be larger than $\sigma$. However, since autocorrelation is taken account in the new model, the estimated standard error of $\varepsilon_n$, $\sigma / \sqrt{(1 - \rho^2)}$, is slightly smaller than the one derived under the i.i.d. assumption. And this is the reason why the estimated welfare cost of business cycles is relatively lower under the AR(1) assumption. Table 1 demonstrates that although significant autocorrelation exists, the estimated welfare cost of business cycles is about 0.7 times the level of the value estimated by Lucas's model, due to a smaller estimated value of $\sigma / \sqrt{(1 - \rho^2)}$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>AR(1) assumption: $\rho=0.693485$. $\sigma=0.033764$. $\sigma / \sqrt{(1 - \rho^2)} = 0.0468636$</th>
<th>i.i.d. assumption: $\sigma_c=0.055172$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001098</td>
<td>0.001523</td>
</tr>
<tr>
<td>5</td>
<td>0.005490</td>
<td>0.007639</td>
</tr>
<tr>
<td>10</td>
<td>0.010981</td>
<td>0.015336</td>
</tr>
<tr>
<td>20</td>
<td>0.021962</td>
<td>0.030910</td>
</tr>
</tbody>
</table>

Table 2 gives the welfare cost of reduced growth in China under the i.i.d. and AR(1) assumptions. The regression results are different due to the modification of the new model. Compared to 0.06 given by Chen (2005), the estimated consumption growth rate is 0.066777 and increases nearly 0.7 percentage points under the AR(1) assumption. This change does not significantly affect the estimated welfare cost of reduced growth. Even though the growth rate is reduced by 5 percentage points and $\gamma$ is equal to 20, the disparity between values given by different models is still less than 0.02.

Also, the calculation results present some notable features. Table 2 illustrates that two assumptions give the same solutions when the relative risk-aversion coefficient $\gamma$ is equal to 1; the gap between calculated values under different assumptions increases with $\gamma$. This gap is also affected by the value of $\mu_c$: given an initial growth rate $\mu$ unchanged, the smaller of $\mu$, i.e. the higher the growth rate is reduced, the larger the gap between calculations using different assumptions.
A Welfare Analysis of the Predictability of Business Cycles in China

Table 2: Comparison between the Estimated Welfare Costs of Reduced Growth under Different Assumptions

<table>
<thead>
<tr>
<th>μi</th>
<th>γ=1</th>
<th>γ=5</th>
<th>γ=10</th>
<th>γ=20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.056777</td>
<td>0.209249</td>
<td>0.029230</td>
<td>0.012072</td>
<td>0.004184</td>
</tr>
<tr>
<td>0.046777</td>
<td>0.462284</td>
<td>0.064817</td>
<td>0.027172</td>
<td>0.009759</td>
</tr>
<tr>
<td>0.036777</td>
<td>0.768268</td>
<td>0.109490</td>
<td>0.046714</td>
<td>0.017441</td>
</tr>
<tr>
<td>0.026777</td>
<td>1.138277</td>
<td>0.168054</td>
<td>0.073326</td>
<td>0.028609</td>
</tr>
<tr>
<td>0.016777</td>
<td>1.585710</td>
<td>0.250092</td>
<td>0.112781</td>
<td>0.046449</td>
</tr>
</tbody>
</table>

Table 2 (continued):

<table>
<thead>
<tr>
<th>μi</th>
<th>γ=1</th>
<th>γ=5</th>
<th>γ=10</th>
<th>γ=20</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d. assumption: β=0.95, μ=0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.209249</td>
<td>0.032676</td>
<td>0.013895</td>
<td>0.005053</td>
</tr>
<tr>
<td>0.04</td>
<td>0.462285</td>
<td>0.073248</td>
<td>0.031657</td>
<td>0.011930</td>
</tr>
<tr>
<td>0.03</td>
<td>0.768267</td>
<td>0.125600</td>
<td>0.055389</td>
<td>0.021728</td>
</tr>
<tr>
<td>0.02</td>
<td>1.138276</td>
<td>0.197131</td>
<td>0.089434</td>
<td>0.036804</td>
</tr>
<tr>
<td>0.01</td>
<td>1.585710</td>
<td>0.304552</td>
<td>0.145194</td>
<td>0.063951</td>
</tr>
</tbody>
</table>

Figure 1 shows the comparison between welfare costs derived from different models. We can see that the welfare cost of reduced growth decreases quickly as the relative risk-aversion coefficient γ goes up while the welfare cost of business cycles increases with γ. In addition, after taking into account the autocorrelation of disturbance, the welfare cost of business cycles becomes slightly lower and about 0.7 times the level of the values estimated by Lucas’ model. The welfare cost of reduced growth does not result in significant changes either. Figure 1 shows that the curves marked by triangles and crosses almost coincide with each other.

According to the i.i.d. assumption, Chen (2005) compared two kinds of welfare costs in China and found that they are approximately equal to each other when the relative risk-aversion coefficient is 10. This result is quite different from Lucas’s view because the welfare cost of reduced growth in the United States is much higher than that of business cycles. Therefore, Lucas’s conclusion that the government should be more concerned with economic growth and less concerned with economic fluctuations is applicable in the United States but not applicable in China where these two costs are roughly equal to each other.

After generalizing Lucas’s model by incorporating autocorrelation into our framework, it is obvious that the AR(1) assumption does not change the calculation results very much, although the estimated costs are both lower than in the case of the i.i.d. assumption. Therefore, the positions of curves that represent the welfare cost do not change dramatically. Also, since these two curves both shift downward, they intersect each other when γ is approximately 10.5, which is slightly larger than in the i.i.d. case. This result implies that the estimated welfare cost of business cycles
and reduced growth are still roughly equal to each other after considering the autocorrelation of the disturbance series, and the main differences from the i.i.d. case are lower estimated values of both welfare costs.

![Graph showing welfare cost of business cycles under different assumptions](image)

Figure 1: Welfare Cost of Business Cycles and Reduced Growth under Different Assumptions

## 5. Concluding Remarks

Based on Lucas's method to measure welfare cost business cycles and reduced growth, this paper develops a more generalized model which assumes that the stochastic disturbance of the consumption stream is an AR(1) process. Under this more reasonable assumption, we re-calculate these two kinds of welfare cost and find that the expression of the welfare cost deduced by conditional expectation is more complicated than that by unconditional expectation. For example, when the model is solved under conditional expectation, factors that can impact the welfare cost of business cycles include not only the relative risk-aversion coefficient and the standard error of the disturbance, but also the subjective time discount factor, the growth rate of consumption and the first-order autoregressive coefficient. But if unconditional expectation is applied to deduce the welfare cost, the final expression is much more simplified. Also, this paper proves that the welfare
A Welfare Analysis of the Predictability of Business Cycles in China

cost of business cycles increases with the first-order autoregressive coefficient of stochastic disturbance of consumption, which means that the stronger the autocorrelation of economic fluctuations, the higher the welfare cost of business cycles. Moreover, although the expression of the welfare cost of reduced growth deducted under conditional expectation is extremely complicated, it reduces to the form derived from the i.i.d assumption when unconditional expectation is applied.

Given the new assumption, the welfare cost of business cycles and reduced growth in China is re-estimated. Compared to calculated results using the i.i.d assumption, the estimated value of welfare cost of business cycles is about 0.7 times the level of the value given by Lucas’s model and the welfare cost of reduced growth is also slightly lower than in the i.i.d case. Therefore, the calculated results derived from the new set of assumptions are not dramatically different, although the AR(1) assumption does gives a more precise theoretic model and econometric estimate. Besides, the numerical simulation shows that the welfare cost of business cycles and reduced growth in China coincide with each other when the relative risk-aversion coefficient is approximately 10.5, which is similar to Chen’s (2005) conclusion. This result implies that these two kinds of welfare cost are roughly equal to each other. Therefore, the Chinese government should focus on both maintaining a high growth rate as well as the stability of the economy.

References


Imrohoroglu Ayse, Selahattin Imrohoroglu, “A Note on the Welfare Cost of Business Cycles and Growth in
Research of Mathematical Economics No. 1 2008


(Responsible Editor: Zhang Lei) (Proofreader: Guo Yumei)


**Labor Supply: A Perspective On The Basis of Social Preference**

Liu Kai  
(School of Economics, Renmin University of China)

**Abstract:** This paper introduces the pioneer work of behavioral economists who break the pure self-interest assumption in neo-classical economics and bring the concept of social preference into the economic analysis; and then constructs a new utility function based on the thoughts of both behavioral economists and Karl Marx. Using this new utility function, this paper sets a new model of labor supply. This model helps us find two important effects in the labor market: microeffect of social consistency and macroeffect of social consistency. At last this paper tries to use this model to explain the strange phenomenon that the wage of farmer-workers of China changes little at a low level and even decreases sometimes with the fast growth of China’s economy.

**Key words:** social preference, microeffect of social consistency, macroeffect of social consistency

**JEL classification:** D01, J22

1. **Introduction**

More and more economic phenomena conflict with the pure self-interest assumption of neo-classical economics. Even if they are of the main stream school, some economists have explained many real-life economic behaviors, deviating from the pure self-interest tradition, for a long time. Becker used to employ the concept of the altruism to explain the voluntary supply of charitable donation and public goods; Selten discussed people’s decision-making behaviors according to a lot of experimental cases and concluded that people’s behaviors are deeply influenced by the thought of equity; Arrow, Samuelson and Sen-Amartya, these three Nobel Prize winners, all pointed out that people’s self-interest is limited and people always care about others’ interests and the equity of the distribution of material benefits (Dong Zhiqiang, 2006). However, the fundamental shock to this basic assumption is the evidence which comes from some experimental economic researches. Based on this, these researches also speed the foundation and development of behavioral economics and experimental economics.

A classical case is the experimental study of the ultimatum game. In order to give the
Research of Mathematical Economics No. 1 2008

Experimental result a good explanation, economists introduce the concept of social preference or equity preference to construct several new models. The so-called “social preference” means that decision makers care about not only their own benefits, but also others’ benefits; the utility of decision makers depends on both their own and others’ benefits. There are two typical models of this kind: one is Charness and Rabin’s model, which brings the absolute difference of the income distribution into the utility function; the other is Bolton and Ockenfels’s model, which brings the relative difference of the income distribution into the utility function. These models also provide new tools and methods for studying other problems.

Actually, Adam Smith, Karl Marx and Hayek had all studied the sociality of human beings and pointed out some profound results. Specially, Marx said: human beings have two dimensions—natural attributes and social attributes. On one hand, human beings are natural beings, which means people need to fulfill their biologic instincts. In an economic society, this is reflected by the fact that people always tend to maximize their own material benefits. On the other hand, human beings are social beings, which means people live in a certain kind of social environment and they have a social language to communicate with each other, they have morality, they have so many kinds of social relationships, etc. It is only one aspect of human beings’ sociality that people care about others’ benefits and the equity of income distribution. Namely sociality is a broader concept than social preference. To make things easy, we only consider social preference here.

This paper attempts to construct a new labor supply model by introducing people’s social preference into utility function, based on Charness and Rabin’s researches and Marx’s thought.


According to the analysis in the introduction part, we redefine individual utility function as follows:

\[ U = u(c, l) + v(c - m_1, l - m_2) \]  \hspace{1cm} (1)

where \( U \) represents utility; \( c \) and \( l \) represent individual consumption and leisure respectively; \( m_1 = \bar{c} \) and \( m_2 = \bar{l} \) represent this individual’s expectation of the society’s average level of consumption and leisure respectively; \( c - m_1 \) and \( l - m_2 \) represent respectively relative level of consumption and leisure. Both \( u \) and \( v \) are increasing concave
functions.

Actually we divide total utility \( U \) into two parts: \( u \) and \( v \). \( u \) represents the degree of happiness of this individual, as a natural being, after consuming goods and enjoying leisure. \( v \) then represents the degree of satisfaction of this individual, as a social being, after comparing his own situation with the average level of the society. As you see, we have made an assumption that the rule of diminishing marginal utility still holds here, which means:

\[
\begin{align*}
u_1 &= \frac{\partial u}{\partial c} > 0, \quad \nu_2 = \frac{\partial u}{\partial l} > 0, \quad \nu_{11} = \frac{\partial^2 u}{\partial c^2} < 0, \quad \nu_{22} = \frac{\partial^2 u}{\partial l^2} < 0; \\
v_1 &= \frac{\partial v}{\partial (c - m)} > 0, \quad \nu_2 = \frac{\partial v}{\partial (l - m)} > 0, \quad \nu_{11} = \frac{\partial^2 v}{\partial (c - m)^2} < 0, \quad \nu_{22} = \frac{\partial^2 v}{\partial (l - m)^2} < 0.
\end{align*}
\]

We also assume that \( \nu_{12} = \nu_{21} > 0 \) and \( \nu_{13} = \nu_{31} > 0 \). This assumption can guarantee that indifference curve is convex to the origin (which means the principle of diminishing marginal rate of substitution holds) and we can do indiffERENCE analysis with diagrams. This assumption is on the basis of reality as well. In fact, we need both consumption and leisure, either of which cannot be lacked of. We will be happy when there is an increase in consumption; but if leisure increases at the same time, we will be happier. In other words, \( \nu_{12} = \frac{\partial^2 u}{\partial c \cdot \partial l} > 0 \). Similarly, we seek for higher relative consumption and also more relative leisure at the same time, which means: \( \nu_{13} > 0 \).

Please notice that if we regard \( m_1 \) and \( m_2 \) as constants, the total utility \( U \) can be expressed in this way: \( U = U(c, l) \), and it has several properties as follows:

\[
\begin{align*}
\frac{\partial U}{\partial c} &= u_1 + v_1 > 0, \quad \frac{\partial U}{\partial l} = u_2 + v_2 > 0; \quad \frac{\partial^2 U}{\partial c^2} = u_{11} + v_{11} < 0, \quad \frac{\partial^2 U}{\partial l^2} = u_{22} + v_{22} < 0; \\
\frac{\partial^2 U}{\partial c \cdot \partial l} &= u_{12} + v_{12} > 0.
\end{align*}
\]

Then \( U \) is the utility function of the neoclassical model. In other words, the neoclassical model of labor supply is only a special case of this model. But the problem is whether \( m_1 \) and \( m_2 \) are constant or not. Please ask yourself and look at the guys nearby.

Then we begin to consider the budget constraint. We have:

\[
I_0 + w(T - l) = c \tag{2}
\]

where \( I_0 \) is the initial real wealth; \( w \) is the real wage rate; \( T \) is the total available time;
Research of Mathematical Economics No. 1 2008

\( l \) is the level of leisure; \( T-l \) is the work time; \( c \) is the level of consumption.

We assume that \( I_0 \) and \( T \) are constants. Consider (1) and (2) simultaneously. Then the problem becomes: \( \max_{(c,l)} U = u(c,l) + v(c-m_1,l-m_2) \) s.t. \( I_0 + w(T-l) = c \)

Construct Lagrangian function:

\[
L = u(c,l) + v(c-m_1,l-m_2) + \lambda \left[ I_0 + w(T-l) - c \right]
\]

So first order conditions are:

\[
\begin{align*}
    u_1 + v_1 &= \lambda \\
    u_2 + v_2 &= \lambda w \\
    I_0 + w(T-l) &= c
\end{align*}
\]  

Second order condition is:

\[
\Delta = w^2 \cdot A - 2w \cdot B + D < 0 
\]  

(\( A = u_{11} + v_{11} \), \( B = u_{12} + v_{12} \), \( D = u_{22} + v_{22} \)).

We know that \( u_{11} < 0, v_{11} < 0; u_{12} > 0, v_{12} > 0; u_{22} < 0, v_{22} < 0 \), therefore \( A < 0, B > 0, D < 0 \), so \( \Delta < 0 \), the inequality (4) holds.

According to the first order conditions, we can get that:

\[
\begin{align*}
    c &= c(w, m_1, m_2) \\
    l &= l(w, m_1, m_2) \\
    \lambda &= \lambda(w, m_1, m_2)
\end{align*}
\]

**Proposition 1:** The individual labor supply \( H = T-l \) is an increasing function of the real wage rate \( w' \).

Proof: Take total differential of (3), we can get:

\[
\begin{align*}
    u_1 dc + u_{11} dl + v_1 (dc - dm_1) + v_{11} (dl - dm_2) &= d\lambda \\
    u_2 dc + u_{12} dl + v_2 (dc - dm_1) + v_{12} (dl - dm_2) &= \lambda dw + wd\lambda \\
    (T-l) dw + w (-dl) &= dc
\end{align*}
\]  

Assume \( dm_1 = dm_2 = 0 \), and substitute them into equation group (5). Then we can get:
Labor Supply: A Perspective On The Basis of Social Preference

\[
\begin{align*}
\left( u_{i1} + v_{i1} \right) \left( \frac{\partial c}{\partial w} \right) + \left( u_{i2} + v_{i2} \right) \left( \frac{\partial l}{\partial w} \right) - \frac{\partial \lambda}{\partial w} &= 0 \\
\left( u_{i2} + v_{i2} \right) \left( \frac{\partial c}{\partial w} \right) + \left( u_{i22} + v_{i22} \right) \left( \frac{\partial l}{\partial w} \right) - w \left( \frac{\partial \lambda}{\partial w} \right) &= 0 \\
\frac{\partial c}{\partial w} + w \cdot \left( \frac{\partial l}{\partial w} \right) &= T - I
\end{align*}
\]

By using Cramer’s Rule, we can get:

\[
\frac{\partial c}{\partial w} = \frac{(T - I) \cdot (D - wB) - \lambda w}{\Delta} \\
\frac{\partial l}{\partial w} = \frac{\lambda + (T - I) (wA - B)}{\Delta} = \frac{\lambda + (T - I) (wA - B)}{\Delta}
\]

Since \( \Delta < 0, T - I > 0, \lambda > 0, A < 0, B > 0, D < 0, w > 0 \), then \( \frac{\partial c}{\partial w} > 0 \), substitution effect \( \frac{\lambda}{\Delta} < 0 \), income effect \( \frac{(T - I) (wA - B)}{\Delta} > 0 \), but the sign of \( \frac{\partial l}{\partial w} \) is ambiguous.

When \( w \) is small, substitution effect dominates income effect, which means \( \frac{\partial l}{\partial w} < 0 \).

When \( w \) is very big, income effect is dominant and \( \frac{\partial l}{\partial w} > 0 \). In general, we assume that \( w \) won’t be so big that income effect will be dominant. In this case, we get an upward sloping labor supply curve as follows (\( H = T - I \) denotes work time):
Until now what we have got is nearly the same as what we can get from the neoclassical model. However, please note that our labor supply curve is related with $m_1$ and $m_2$ as well, namely $H = s = s(w, m_1, m_2)$.

**Proposition 2:** Other things equal, when people’s expectation of the social average level of consumption increases, they will adjust their behaviors to increase their consumption as well; when people’s expectation of the social average level of leisure goes up, they will also increase their leisure to keep up with the whole society.

We call the phenomena above “microeffect of social consistency”.

Proof: From (5) we can get:

$$
\begin{align*}
\frac{\partial c}{\partial m_1} &= \frac{w}{\Delta}(wv_{11} - v_{12}) \\
\frac{\partial c}{\partial m_2} &= \frac{w}{\Delta}(wv_{12} - v_{22}) \\
\frac{\partial l}{\partial m_1} &= \frac{1}{\Delta}(v_{12} - wv_{11}) \\
\frac{\partial l}{\partial m_2} &= \frac{1}{\Delta}(v_{22} - wv_{12})
\end{align*}
$$

We know that $\Delta < 0$, $w > 0$, $v_{11} < 0$, $v_{12} > 0$, $v_{22} < 0$, thus:

$$
\frac{\partial c}{\partial m_1} > 0, \quad \frac{\partial c}{\partial m_2} > 0, \quad \frac{\partial l}{\partial m_1} > 0, \quad \frac{\partial l}{\partial m_2} > 0
$$

(6) 
(7)

The inequalities (6) and (7) reflect the result of proposition 2.
**Labor Supply: A Perspective On The Basis of Social Preference**

We can make a summary of all the contents above that \( I = I(w, m, m) \) and \( \frac{\partial I}{\partial w} < 0 \),

\[
\frac{\partial I}{\partial m_1} < 0, \quad \frac{\partial I}{\partial m_2} > 0; \quad \text{labor supply} \quad s = T - I = s(w, m_1, m_2) \quad \text{and} \quad \frac{\partial s}{\partial w} = -\frac{\partial I}{\partial w} > 0
\]

\[
\frac{\partial s}{\partial m_1} = -\frac{\partial I}{\partial m_1} > 0, \quad \frac{\partial s}{\partial m_2} = -\frac{\partial I}{\partial m_2} < 0.
\]

After studying individual labor supply, we begin to analyze labor supply in the whole market.

First of all, we make assumptions as follows: (1) The number of labor suppliers in the market is \( N \); (2) They have the same expectation of the social average level of consumption and leisure, which are \( m_1 \) and \( m_2 \) respectively; (3) The labor supply function of supplier \( i \) is \( s_i = s_i(w, m_1, m_2) \) \((i = 1, 2 \ldots N)\).

Then the market labor supply function is \( S = \sum_{i=1}^{N} s_i(w, m_1, m_2) \), and

\[
\begin{align*}
S_1 &= \frac{\partial S}{\partial w} = \sum_{i=1}^{N} \frac{\partial s_i}{\partial w} > 0 \\
S_2 &= \frac{\partial S}{\partial m_1} = \sum_{i=1}^{N} \frac{\partial s_i}{\partial m_1} > 0 \\
S_3 &= \frac{\partial S}{\partial m_2} = \sum_{i=1}^{N} \frac{\partial s_i}{\partial m_2} < 0
\end{align*}
\]

When people’s expectation of the social average level of consumption increases from \( m_1 \) to \( m'_1 \), then the market labor supply will also increase and the supply curve will move to the right side. This effect is determined by microeffect of social consistency. Similarly, when the expected social average level of leisure increases from \( m_2 \) to \( m'_2 \), the market labor supply will decrease and the supply curve will move to the left side (please see Graph 2).
3. Labor Market under Rational Expectation

Now the following part should focus on the whole labor market. Suppose the initial supply curve is \( S = S(w, m_1, m_2) \), and the demand curve is \( D = D(w) \) (please see Graph 3).

Then the market wage rate and the social total work time will be deter- mined by the equation \( S(w, m_1, m_2) = D(w) \), and each supplier will have his corresponding consumption \( c_i \) and leisure \( l_i \). Thus the social average consumption \( \bar{c} \) equals \( \left( \sum_{i=1}^{N} c_i \right) / N \) and average leisure \( \bar{l} \) equals \( \left( \sum_{i=1}^{N} l_i \right) / N \).

But it’s not the final case because \( m_1 \) is not necessarily equal to \( \bar{c} \) and \( m_2 \) doesn’t necessarily equal \( \bar{l} \) either. As long as \( (m_1 - \bar{c})^2 + (m_2 - \bar{l})^2 \neq 0 \), every labor supplier will change their initial expectation, change their utility function and finally change the market supply function to \( S = S(w, \bar{c}, \bar{l}) \). However, this is the result neither because one of \( \bar{c} \) and \( \bar{l} \) is still not necessarily equal to the social average level. Then everybody will adjust again. This kind of adjustment will last until the expected values equal the real ones exactly.
Now we are going to analyze this problem under the assumption of rational expectation. First, let us make several assumptions as follows:

1. There are \( N \) labor suppliers in the market, and the average initial wealth is \( I_0 \).
2. Every labor supplier has the same expectation of the social average level of assumption and leisure, and the expected values are exactly the real values \( \bar{c} \) and \( \bar{l} \).

Therefore, labor supply function is (see Graph 4)

\[
S = S(w; \bar{c}, \bar{l})
\]  

Now, the social average leisure is

\[
\bar{l} = T - \frac{H_0}{N}
\]  

The social average consumption is

\[
\bar{c} = \frac{N \cdot I_0 + w_0 \cdot H_0}{N} = I_0 + \frac{w_0 \cdot H_0}{N}
\]

We know that \( H_0 = D(w_0) \). Substitute it into (10) and (11) and get:

\[
\bar{l} = T - \frac{D(w_0)}{N}
\]
Then substitute (12) and (13) into function (9) and get:

\[ S = S(w, I_0 + \frac{w_0 \cdot D(w_0)}{N}, T - \frac{D(w_0)}{N}) \]

In other words, when the labor supply function is

\[ S = S(w, I_0 + \frac{w_0 \cdot D(w_0)}{N}, T - \frac{D(w_0)}{N}) \]

the supply curve will be intersectant with the demand curve at the point \((w_0, H_0 = D(w_0))\). At this moment, the expected values equal the real ones, and the market comes to the state of equilibrium, when

\[ S = S(w_0, I_0 + \frac{w_0 \cdot D(w_0)}{N}, T - \frac{D(w_0)}{N}) = D = D(w_0) \]

Namely the equilibrium point \((w_0, H_0 = D(w_0))\) can be viewed as the intersection of the curve \(S(w) = S(w, I_0 + \frac{w \cdot D(w)}{N}, T - \frac{D(w)}{N})\) and the demand curve \(D(w)\).

We define that function \(S(w) = S(w, I_0 + \frac{w \cdot D(w)}{N}, T - \frac{D(w)}{N})\) is labor supply function under rational expectation. Then we can get:
**Proposition 3:** In the labor market of a developing country, labor supply function under rational expectation \( S(w) \) is an increasing function of \( w \).

Proof: \[
\frac{dS(w)}{dw} = S'_1 + \frac{S''}{N} [D(w) + w \cdot D'(w)] - \frac{S'_1}{N} \cdot D(w)
\]

We take notations that: \( E = \frac{S''}{N} \cdot D(w) \), \( F = \frac{S'_1}{N} \cdot w \cdot D'(w) - \frac{S'_1}{N} \cdot D(w) \), \( G = E + F \), then:

\[
\frac{dS(w)}{dw} = S'_1 + G = S'_1 + E + F
\]  \hspace{1cm} (14)

Where \( S'_1 = \frac{\partial S}{\partial w} \) is called “individual effect”, which represents the effect of change in \( w \) on the labor supply under the condition that the expected social average consumption and leisure remain the same. In light of the analysis in part II, we know that \( S'_1 \) can be divided into substitution effect and income effect, and generally substitution effect is dominant. As known from (8); \( S'_1 > 0 \).

\( G = E + F \) is named “social effect”, where \( E \) and \( F \) are called “wage effect” and “demand effect” respectively. \( E \) represents the effect of change in the social average consumption, which is result from change in \( w \), on the labor supply when other things are equal. \( F \) represents the effect of change in the social average consumption and leisure, which is result from change in the labor demand as a result of change in \( w \), on the labor supply, other things equal.

We know from (8) that: \( S'_1 > 0 \), \( S'_3 < 0 \). Because the demand curve is downward sloping, \( D'(w) < 0 \). Thus \( E > 0 \), \( F < 0 \). When \( w \) is relatively low, \( D(w) \) is quite great, and wage effect dominates demand effect, which means \( G > 0 \). When \( w \) is very high, demand effect is dominant, which implies \( G < 0 \). In general, we assume that wage effect is dominant and \( G > 0 \), which will hold more probably in the labor market of a developing country.

Since \( S'_1 > 0 \) and \( G > 0 \), we can get from equality (14) that \( \frac{dS(w)}{dw} > 0 \), implying labor supply function under rational expectation \( S(w) \) is an increasing function of \( w \). So the determination of equilibrium wage could be expressed with Graph 5 below.
The next question is what’s the relationship between $S = S(w; \bar{c}, \bar{l})$ and $S(w) = S(w; l_0 + \frac{w \cdot D(w)}{N}, T - \frac{D(w)}{N})$ (Where $\bar{c} = l_0 + \frac{w_0 \cdot D(w)}{N}$, $\bar{l} = T - \frac{D(w)}{N}$).

According to the assumption of rational expectation, at the point $(w_0, H_0)$, $S(w_0)$ equals $S(w_0, \bar{c}, \bar{l})$. But $\frac{dS(w)}{dw} = S_i + G$, $\frac{dS(w, \bar{c}, \bar{l})}{dw} = S_i$ and $G > 0$, so $\frac{dS(w)}{dw} > \frac{dS(w, \bar{c}, \bar{l})}{dw}$. Thus when $w > w_0$, $S(w) > S(w, \bar{c}, \bar{l})$; and when $w < w_0$, $S(w) < S(w, \bar{c}, \bar{l})$. Then we can describe this relationship simply in the following way:

$S(w) \approx S(w, \bar{c}, \bar{l}) + \alpha(w - w_0)(\alpha > 0)$, which can be reflected graphically below:
**Labor Supply: A Perspective On The Basis of Social Preference**

In the light of the analysis above, we can see clearly that:

**Proposition 4:** Labor supply is intimately related with labor demand. This is an important feature of labor force commodity and labor market, compared with other commodities and markets. However, they are absolutely independent in neoclassical model.

### 4. Macroeffect of Social Consistency

After studying supply, demand and the determination mechanism of equilibrium, naturally we are going to examine the influence of the movements of supply and demand on the labor market and its equilibrium.

**Proposition 5:** Labor supply will increase to meet the increased demand when labor demand increases. Similarly, when labor demand decreases, labor supply will decrease as well.

We call this phenomenon “macroeffect of social consistency”.

Proof: Suppose when the wage rate is the same as before, labor demand increases, which is shown in Graph 7: demand curve $D_i(w)$ moves rightward to $D_i'(w)$.

Then will labor supply curve under rational expectation move?

As $S(w) = S(w', l_0 + \frac{w \cdot D(w)}{N}, T - \frac{D(w)}{N})$, take partial derivative:
\[ \frac{\partial S(w)}{\partial D} = S_2 \cdot \frac{w}{N} - \frac{S_3}{N} \]

As known from (8): \( S_2 > 0 \) and \( S_3 < 0 \), So \( \frac{\partial S(w)}{\partial D} > 0 \).

Accordingly \( S(w) \) will move rightward as well. See Graph 7: supply curve moves from \( S_1(w) \) to \( S_2(w) \).

The result is that equilibrium point moves from A to B. After comparison, we can see: the wage rate keeps relatively stable, but the social work time increases a lot.

Graph 7  Movement of Equilibrium Point

Similarly when demand curve moves leftwards, supply curve will move upwards.

5. One Application of the Model: An Explanation to the Current Situation of Chinese Farmer-workers’ wage

In China, there are a huge number of farmers coming into cities to become farmer-workers.

Please see Table 1.

| Table 1 | The number of farmer-workers in China |
Labor Supply: A Perspective On The Basis of Social Preference

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No.(10,000)</td>
<td>5066</td>
<td>5556</td>
<td>5635</td>
<td>5984</td>
<td>6851</td>
<td>7550</td>
<td>8961</td>
<td>9431</td>
<td>9820</td>
<td>10260</td>
<td>10824</td>
</tr>
</tbody>
</table>

Data Source: 《The Outworkers’ Employment and Training of Rural Labor Force》，Zhao Changbao (Deputy Director of The Rural Economic Research Center of The department of Agriculture).

How about their wage and work time? Statistical data shows that the average wage of farmer-workers in Shenzhen in 2001 is 588 Yuan/month, which is lower than the level in 1980s (Lu Xueyi, 2005). This wage level, 588 Yuan/month, roughly coincides with the average wage -7122 Yuan/year-for workers in Shenzhen in 2001（Statistics Bureau of Shenzhen city, 2002）. Although there are differences among statistical calibers (say, somebody else thinks that the average wage of farmer-workers in Shenzhen is about 800 Yuan/month (Zhou Qingjie, 2004)), it is nearly a common sense that the wage of farmer-workers has changed little and even decreased sometimes in the past ten years (Zhou Zhengping, 2003). An investigation from Chongqing in Oct. 2004 shows that the average wage of farmer-workers is only 650 Yuan/month and they work for 10.5 hours everyday on average (Zhou Qingjie, 2004). Investigations from Guangdong and other big cities say almost the same thing.

Why does the real wage of farmer-workers of China change little at a low level and even decrease sometimes with the fast growth of China’s economy?

We at once think of Lewis’ Model: suppose the supply curve of rural surplus labor force is infinitely elastic. With the economic growth, the rural surplus labor force is gradually transferred out to industrial sector, but the wage level remains the same as before. However, this classical model cannot help to explain why the real wage decreases sometimes in this process.

The model of social preference given by this paper could give some convincing explanation: we assume the supply curve of rural surplus labor force is very elastic, but not infinitely elastic. With the fast growth of China’s economy, many firms expand scale and there come out many new factories, which mean the demand for labor force increases and the labor demand curve moves rightwards. Since there exists macroeffect of social consistency, the labor supply curve will also move rightwards. As a result, the social total amount of labor will increase much, but the real wage rate will remain nearly the same or even decrease. The big increase in the social total amount of labor means GDP will grow much. Although the average work time becomes long, the real wage won’t increase much and may even decrease because the wage rate remains nearly the same or
even decrease.

With the growth of economy, the living standard is improving. Even if the wage of farmer-workers does not increase in the city, as social beings with social preference, farmers who have not intended to go to the city before now go to the city to become a farmer-worker; who have already been in the city before now are willing to supply more hours to work. Only in this way they can increase their incomes to keep up with the whole society. However, just in this way their wage level remains nearly the same, and even decreases sometimes.

6. Concluding Remarks

This paper constructs a new labor supply model by introducing social preference into utility function, and then finds two important effects in the labor market: microeffect of social consistency and macroeffect of social consistency. Then we try to use this model to explain the situation that the wage of farmer-workers of China changes little at a low level and even decreases sometimes with the fast growth of China’s economy. Of course many improvements can be made about this paper: one can divide the labor market into two parts (farmer-workers market and the other market) to do a more accurate analysis; one can bring the minimal wage into the model; one can discuss the welfare at the equilibrium, analyze the difference between the equilibrium and pareto optimality, and then give some suggestions to the government and labor union, etc.

References


**Labor Supply: A Perspective On The Basis of Social Preference**


(Responsible Editor: Ai Yi) (Proofreader: Guo Yumei)
A Study on the Difference among the Welfare Costs of Chinese Inflation

Zhou Xuan
(School of Economics, Renmin University of China)

Abstract: This paper discusses the welfare costs of China’s inflation and the difference of welfare costs through the Bailey’s theoretical model based on Chinese urban and rural income data. The major findings are: (1) The welfare costs of China’s inflation of urban households are about five to six times higher than that of rural households for both log-log demand curve and semi-log demand curve. (2) More attention should be paid to low-income group in urban areas. Therefore, while economic growth is highly emphasized, economic inflation should be paid attention to and all sorts of differences of inflation welfare cost also should be taken into account.

Key words: Inflation, Welfare cost, Difference

1. Introduction

Inflation refers to a sustaining rise of the national economy’s general level of prices of goods and services over a period of time. According to the classical monetary theory, individual economic welfare depends on relative commodity price; therefore the increase in the price level seems no threat to individual economic welfare. Obviously, that is not true. Research indicates that inflation is the most common word in US newspapers and price stability is all along a primary goal of monetary policy of each government.

Why could inflation make such wide impact on economy? Because the sustaining rise in price motivates people to change their behaviors and the distortion brought about by this motivation leads to the social deadweight loss, which is called the welfare cost of inflation, such as heel cost, menu cost, relative price change, tax revenue distortion, and the wealth is reallocated etc.

There have been many relevant researches of welfare cost of inflation domestically and abroad, Chen Yanbin and Ma Lily (2007) carries on systematic retrospect and summary of these researches, including consumers' surplus method, general equilibrium model of neoclassical macroeconomics and new Keynes' doctrine macroeconomics general equilibrium model.

However, the above research models all implied at least two assumptions: "Commodity
A Study on the Difference among the Welfare Costs of Chinese Inflation

basket" hypothesis and "overall benefit" analysis method. Specifically, the "commodity basket" hypothesis refers to that the inflation rate is one kind of average results. For example, the consumer price index (CPI) commonly used by economists is calculated by averaging the price change of typical family purchase commodity and service. "Overall benefit" analysis method refers the research tacitly approve that nationals’ response to the price change is consistent, and no heterogeneity exists. The assumptions simplify the research of the welfare cost of inflation and produce a basic answer to the issue.

Actually, the above assumptions do not stand in the real life. First, inflation may be caused by different rise in all commodities and services, as well as caused by great rise in some commodities and light fall in others. Second, price changes of different commodities and services influence different consumers, for discrepancy of asset structure and consumption habit among different consumers. Therefore, statistically average may not reflect the real welfare cost of inflation.

Since different people may suffer differently from inflation, is the difference important? The economists of Federal Reserve Bank of New York - Bart Hobijn and David Lagakos (2003) tried to answer this question. They surveyed the US CPI data of 1987-2001, and studied the inequality of inflation in different social groups. The authors indicate that the inflation the elderly faced is about 0.2 to 0.4 percent higher than the average, if US pensions are indexed, OASI will go bankrupt five years earlier.

On the face of it, it is very necessary for us to research the difference among welfare costs of inflation. Actually, there are already many researches on the relation between inflation and benefits distribution. Earlier work by Michael (1979) and Hagemann (1982) has focused on the differences in changes in the cost of living across U.S. households. Later studies, like Amble and Stewart (1994), Garner et al. (1996), Idson and Miller (1999), and Hamilton (2001) have focused on the inflation experience of particular groups. Erosa and Ventura (2002) thought heterogeneity commonly existed. They assumed two families – rich one and poor one, two types of trade – cash and credit in their family heterogeneity model. And they thought the rich family used credit more often and suffered less inflation loss. They used education level to identify two kinds of families, and calculated that the welfare cost of inflation of low income family is about two times of the high income one. Cysne (2006) theoretically discussed the family heterogeneity of inflation welfare cost in Shopping Time Model. In the model, families of different productivities and different transaction efficiencies are studied. Cysne called it the Intra - Household Model (IH
model). Cysne (2006) has compared his result with the traditional Shopping Time Model and IH model, and concluded that the IH model can bring about different money demand functions, thus different inflation welfare cost.

This paper will sequentially break the "commodity basket" hypothesis and "overall benefit" analysis method, adopting urban and rural data to estimate respectively the welfare costs of inflation and study the heterogeneity of inflation costs between urban and rural areas in China. I would like to present my paper as follows: In section two I conduct a introduction to the consumers' surplus method; Section three adopts the consumers' surplus method mentioned in section two to estimate the welfare cost of inflation in China real example; Section four is the conclusion and future directions.

2. Consumers' Surplus Method

Friedman's optimum monetary quantity rule says, in a monetary economy, in order to guarantee the efficiency of resource distribution by perfect competition, the nominal interest rate must equal to zero, at which point the distortion inflation causes is minimum. So the inflation rate which makes the nominal interest rate zero should be the optimum inflation rate. If the inflation rate in one economy exceeds the optimum inflation level, accordingly the nominal interest rate exceeds zero, it is not Pareto optimum for monetary distribution, the social welfare drops and the welfare cost of inflation comes into existence. The welfare cost of inflation defined by Bailey (1956) is the square measure under money demand curve when the nominal interest rate rises from 0 to i, accordingly from the optimum inflation level to a certain inflation rate, the area of function curve of monetary demand, sometimes this area is also called "the welfare triangle".

Let demand function for real balances take the form $M_r / P_r = m_r \equiv L(i_r, y_r)$, where $M_r$ represents nominal balances, $P_r$ represents price level, $m_r$ represents real balances, $i_r$ is real interest rate and $y_r$ is real consumption. Figure 1 shows money demand and money supply curve equilibrium, downward demand curve and vertical supply curve. The ordinate of this figure represents the nominal interest rate, and the abscissa represents the real balances. The nominal interest rate is the opportunity cost for a consumer who chooses to hold currency instead of deposit, 

---

1. Because nominal interest rate = real interest rate + inflation rate, so when the nominal interest rate is zero, the optimum inflation rate is negative.
A Study on the Difference among the Welfare Costs of Chinese Inflation

and this can be interpreted as the price of currency. Consumers’ surplus is the total income from holding currency deducting overall cost, expressed by the area under demand curve and above price level. When the nominal interest rate equals zero, the opportunity cost of holding money is also zero and consumers’ surplus reaches maximum. When the nominal interest rate rises to i, the opportunity cost of holding money also rises and this will cause net losses of consumers’ surplus, indicated by the shaded area and rectangle Q in the figure. On the other hand, government as money producer, its producers’ surplus is also the total income from publishing money deducting overall cost, expressed by the area between under price level and above supply curve. When the nominal interest rate rises to i, the production cost of publishing money can be omitted, and producers’ surplus increases and forms the public revenue, indicated by the area of rectangle Q in the figure. The total reduction of consumers’ surplus and producers’ surplus forms meaningless losses, and such meaningless loss is the welfare cost of inflations, measured by the shaded area under the inverse money demand function.

Let \( m(i) \) be the estimated money demand function, and \( \phi(m) \) be the inverse function of \( m(i) \), then the deadweight loss can be defined as:

\[
\text{deadweight loss} = \int_{m(i)}^{m(0)} \phi(m) dm = \int_0^i m(x) dx - im(i)
\]

Here the deadweight loss is measured by real balances, and the welfare cost of inflation is further defined as the ratio of deadweight loss to consumptions: \(^2\)

\[
\nu(i) = \frac{\text{deadweight loss}}{y_i} = \frac{\int_0^i m(x) dx - im(i)}{y_i}
\]

(1)

From (1) we can find that the calculation of welfare cost of inflation relies on certain form of money demand function; the most popular money demand curves are semi-log curve and log-log curve.

Assuming the income elasticity of money demand is 1. \(^3\)Cagan (1956) and Bailey (1956) use semi-log money demand function \( m(i) = Be^{-\xi i} y_i \) to estimate the welfare cost of inflation:

---

\(^2\) Some literatures write the welfare cost of inflation as the ratio of deadweight loss to the income, but throughout this paper, we write it as the ratio of deadweight loss to consumption. Firstly, because the dominating variable of households is consumption instead of income, using consumption can make the model easier; secondly, if we assume market equilibrium, then consumption equals income theoretically.

\(^3\) Lucas (2000) observes the time series of the ratio of M1 to nominal GDP, for the period of 1900-1994 in United States, and finds the money-income ratio is essentially trendless and he concludes that the money demand elasticity of income is zero. Marty (1999) also believes that it is important to assume a zero elasticity, otherwise, researches cannot be conducted under steady state.
Research of Mathematical Economics No. 1 2008

\[
W(t) = \left( \int_0^t B e^{-\xi t} \, dt - iB e^{-\xi t} \right) = \frac{B}{\varepsilon} [1 - (1 + \varepsilon t) e^{-\varepsilon t}] 
\]

Lucas (2000) chooses log-log money demand function \( m(i) = A i^{-\eta} y_i \) to estimate the welfare cost of inflation:

\[
W(t) = \left( \int_0^t A x^{-\eta} \, dx - iA i^{-\eta} x \right) = \frac{A \eta}{1 - \eta} \int_0^t x^{1-\eta} \, dx 
\]

3. Empirical Study of the Welfare Cost of Inflation of China

3.1 Parameter Estimation

In this part, we use quarterly data to estimate the economic parameters and behavior parameters needed for welfare cost calculation in urban and rural China. To estimate the money demand curve, we should use the data of money supply, income and interest rate of urban and rural respectively.

First, I define money supply as \( M_2 \). Since it is impossible to get the \( M_2 \) data directly for urban and rural households, I assume that for urban and rural households, their savings \( S \) is a constant ratio of \( \alpha \) to the money supply. Thus, we have \( M_2 = S / \alpha \).

\( \alpha \) is calculated from national data. In order to fully reflect the supply-demand information of money, we choose the weighted average of inter-bank interest rate (seven days) as the nominal interest rate. The data of saving deposits and incomes from 2001 to 2005 comes from China Monthly Economic Indicators. Inter-bank interest rate is from the website of China Central Bank.

For money demand function, we choose log-log demand curve and semi-log demand curve to estimate the welfare cost of inflation. For log-log demand curve \( m(i) = A i^{-\eta} y_i \), we have:

\[
\log \frac{m(i)}{y_i} = \log A - \eta \log i 
\]

For semi-log demand curve \( m(i) = B e^{-\varepsilon i} y_i \), we have:

\[
\log \frac{m(i)}{y_i} = \log B - \varepsilon i 
\]

Conduct OLS estimation to (4) and (5), see results below:

---

4 In the estimation of money demand function, dependent variable is \( m / y = (M / P) / (Y / P) = M / Y \), so nominal money supply and social income needn’t be adjusted by inflation.
A Study on the Difference among the Welfare Costs of Chinese Inflation

To urban households, for the log-log demand curve, \( \eta = 0.18 \), \( A = 0.82 \); for the semi-log demand curve, \( \varepsilon = 8.22 \) and \( B = 1.94 \).

To rural households, for the log-log demand curve, \( \eta = 0.35 \), \( A = 0.04 \); for the semi-log demand curve, \( \varepsilon = 18.59 \) and \( B = 0.24 \).

3.2 Calculation of Welfare cost of Inflation of China

From part 1 we can get two demand curves estimated. Specifically, for log-log demand curve, urban \( m(i) = 0.82i^{-0.18}y_i \) and rural \( m(i) = 0.04i^{-0.35}y_i \), we can calculate the welfare cost \( n(i) \):

For urban

\[
n(i) = \frac{A\varepsilon}{1-\eta} i^{\eta-1} = 0.18 i^{0.82}
\]

For rural

\[
n(i) = \frac{A\varepsilon}{1-\eta} i^{\eta-1} = 0.02 i^{0.65}
\]

As for semi-log demand curve, urban \( m(i) = 1.94e^{-8.22i}y_i \), rural \( m(i) = 0.24e^{-18.59i}y_i \), the welfare cost \( n(i) \) is:

For urban

\[
n(i) = \frac{B}{\varepsilon}[1-(1+\varepsilon i)e^{-\varepsilon i}] = 0.24[1-(1+8.22i)e^{-8.22i}]
\]

For rural

\[
n(i) = \frac{B}{\varepsilon}[1-(1+\varepsilon i)e^{-\varepsilon i}] = 0.01[1-(1+18.59i)e^{-18.59i}]
\]

Table 1 lists the welfare costs at different nominal inflation rate. We can find with both log-log demand curve and semi-log curve, the urban welfare cost estimated is greater than that of rural households at the same nominal inflation rate. For example, at three percent nominal interest rate, the urban welfare cost is about 0.8 percent higher than rural one with Log-log demand curve and 0.5 percent higher with semi-log demand curve.

<table>
<thead>
<tr>
<th>Nominal interest rate (the ratio of income)</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>urban</td>
<td>0.004</td>
<td>0.007</td>
<td>0.010</td>
<td>0.013</td>
<td>0.018</td>
<td>0.023</td>
<td>0.027</td>
</tr>
</tbody>
</table>
Research of Mathematical Economics No. 1 2008

<table>
<thead>
<tr>
<th></th>
<th>rural</th>
<th>0.001</th>
<th>0.001</th>
<th>0.002</th>
<th>0.002</th>
<th>0.003</th>
<th>0.004</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-log demand curve</td>
<td>urban</td>
<td>0.001</td>
<td>0.003</td>
<td>0.006</td>
<td>0.010</td>
<td>0.021</td>
<td>0.034</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>rural</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
<td>0.004</td>
<td>0.005</td>
</tr>
</tbody>
</table>

For comparison, I list the national welfare cost of inflation from Zhou Xuan and Yang Fan (2007) below.

Table 2 National welfare cost calculated by consumers’ surplus method

<table>
<thead>
<tr>
<th>Nominal interest rate</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>5%</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W' ) (the ratio of income) Log-log demand curve</td>
<td>0.002</td>
<td>0.004</td>
<td>0.005</td>
<td>0.008</td>
<td>0.009</td>
<td>0.011</td>
<td>0.013</td>
</tr>
<tr>
<td>( W' ) (the ratio of income) Semi-log demand curve</td>
<td>0.001</td>
<td>0.006</td>
<td>0.012</td>
<td>0.032</td>
<td>0.045</td>
<td>0.075</td>
<td>0.111</td>
</tr>
</tbody>
</table>

Chen Yanbin and Ma Lily (2007) have indicated that Log-log demand curve gives a better fit to China’s money demand curve, thus a better estimation of welfare of inflation. Therefore, I take a further contrast of \( W' \) based on log-log demand curve and find that the national welfare cost in table 2 is always between urban consequence and rural consequence, which means in some degree this estimation of welfare cost of inflation is acceptable and the result can give some implications to economy.

4. Conclusions and Future Directions

Inflation rate is an important indicator to measure whether the macro-economy is running stably and benignly. This article delivers a comparison of inflation welfare cost between urban and rural households. Based on this, traditional consumer surplus method is adopted for the calculation and comparison of the welfare costs of inflation in China. This article yields these main conclusions:

First, in China, extensive difference on welfare cost exists. According to my calculation, the urban welfare cost estimated is averagely five to six times as high as of rural households at the same nominal inflation rate. This result differs a lot from Erosa and Ventura (2002). In US, they conclude that the rich are better able to protect themselves against the effects of inflation than the
**A Study on the Difference among the Welfare Costs of Chinese Inflation**

poor. In particular, the rich and more sophisticated are likely to have better access to financial instruments that hedge in some way against inflation, while the small portfolios of the poor are likely to have a large share of cash. However, that doesn’t tell the real story in China.

In China, financial market hasn’t been fully developed, and financial instruments are not popular among Chinese urban consumers. On the other hand, the urban have a much larger commodity basket than the rural, which means that urban households may suffer inflation at a higher possibility. For example, while urban consumers complain the price upsurge in house, food and other chain reactions caused consequently, the rural consumers have no such harass, since they already have their land and food. Their life is much less sensitive to the market than the urban consumers.

Second, more attention should be paid to low income group in urban areas. This group may cover the most victims of inflation. Walliam Easterly and Stanley Fischer (2001) have pointed out that inflation tax rate has a significant positive effect on the increase in poverty. I think that is mainly ascribed to the increase in urban poverty. Therefore, the government should first take urban poverty into consideration when making anti-inflation policies.

This article is only a first-step study on measuring the impacts of inflation on urban and rural households respectively. The work is flawed in several aspects, which means more aspects are worth exploring. First and foremost, behavior parameters of Chinese families play an important role in the calculation of welfare cost of inflation. But there is no such complete estimation about this contemporarily, so the numerical simulation has no way to show the real story in China. Besides, welfare cost models in this article are all perfect market models, which contradict the reality. The reasons for imperfect market can be asymmetric information, transaction cost, uninsurable special risks, short selling bounds of capital, credit limits of wealth and so on. Those factors will lead to the result that the equilibrium is not Pareto effective; consequently it will make more sense to discuss the welfare cost of inflation in China with imperfect markets. Finally, compared to developed countries, there is obvious dual economy in China, and the mechanism of interest rate marketization has not been well constructed. Under this circumstance, response of price level will lag more and thus be much stickier than developed countries. Neo-classical assumptions of rational expectation and sticky price are integrated in New-Keynesm, and are more and more widely applied to solving macroeconomic problems, thus the framework of new-keynesm will be more compatible in estimating the welfare cost of inflation in China.
Reference


(Responsible Editor: Qiu Zhesheng) (Proofreader: Guo Yumei)
Infrastructure, Public Services and Optimal Economic Growth

Infrastructure, Public Services and Optimal Economic Growth

Huo Zhen

(School of Economics, Renmin University of China)

Abstract: This paper studies the optimal tax rate and the allocation of government resources between infrastructure and public services in an endogenous growth model. The key feature is that the expenditure on infrastructure not only affects the production directly, but also enhances the accumulation of public services. The optimal tax rate equals the government spending’s contribution to the production which is similar to the Barro’s rule. The optimal composition is determined by the technology of production and public services accumulation.

Keywords: Infrastructure, Public Services, Fiscal Policy, Economic Growth

1. Introduction

The effect of government spending on economic growth has always been the subject of analytical research. The first type of research uses AK endogenous growth model to analyze how the government spending affects the economic growth (Arrow and Kurz, 1970; Barro ,1990; Turnovsky and Fisher, 1995). The second type of research focuses on the relationship between human capital accumulation and government spending in an endogenous growth model(Glomm and Ravikumar, 1998). When they investigate the functions of public spending and public goods, the key point is how the public capital affects the production. Barro (1990) assumed that the public investment enhances the production directly. Futagami (1993) treated that the public investment only indirectly affects the production by building up the public capital to stimulate the economy. Greiner and Hanusch (1998) considered the public spending both productive and non-productive.

But these researches usually overlook the effect of composition of government spending on economic growth. Glomm and Ravikumar (1997) first distinguished the two types of public
spending which affect the production in different ways. Agenor (2005) investigated the composition of public spending carefully. In fact, what a government buys may be important to the economic growth rate. When the public spending on a certain item has no effect on production, it will generate negative effects on the economy. We can imagine that there exists a growth maximizing composition of the public spending.

This paper presents a theoretical model based on Agenor’s framework to study the optimal fiscal structure between infrastructure and public services. It extends the previous work in two ways: first, the mechanism of public spending determining production is different; secondly, this paper compares two kinds of utility function (separable and nonseparable).

The rest of the paper proceeds as follows. Section 2 introduces the basic model. Section 3 analyzes the decentralized equilibrium. Section 4 gives the optimal fiscal structure. Section 5 and section 6 study the extended of the basic model.

2. The Basic Model

Consider a closed economy with a perfect competitive market. There is an infinitively-lived representative household who produces a single good, which can be consumed or invested. The government leaves a flat tax on output to maintain a balanced budget. At the same time, the government provides two kinds of public goods—— infrastructure and public services free of charge. We assume that the infrastructure, such as harbors, highways, only enhances productivity of the economy in the form of flow, and the public services, for example, health, education, are both productivity-enhancing and utility-enhancing as a stock.

2.1 Production

The output, \( Y \), is produced with infrastructure, public services and private capital, assuming using a Cobb-Douglas technology for simplicity:

\[
Y = G_t^a E^b K_p^{1-a-b}
\]

where \( G_t \) denotes the infrastructure, \( E \) denotes the stock of public services, \( K_p \) denotes the stock
of private capital. Here, we also assume \( \alpha, \beta \in (0,1) \) and \( \alpha + \beta < 1 \), which are the elasticity of output with respect to the three factors. Thus, the infrastructure, the stock of public services and the stock of private capital all have positive effects on the marginal output. We can rewrite equation (1) as:

\[
Y = \left( \frac{G_I}{K_p} \right)^\alpha \left( \frac{E}{K_p} \right)^\beta K_p
\]  

(2)

which implies that when \( \frac{G_I}{K_p} \) and \( \frac{E}{K_p} \) are constant, the per capital output \( \frac{Y}{K_p} \) will be also constant. On the balanced growth path, the output, the stock of private capital and the consumption will grow at the same rate.

2.2 Household

The rational infinitively-lived household’s object is to maximize the discount stream of utility:

\[
\max_c V = \int_0^\infty U \exp(-\rho t) dt
\]  

(3)

where \( \rho > 0 \) is the exogenous discounted rate. The instantaneous utility is given by consumption and the stock of public services. We first consider the condition that the utility is separable in consumption and public services, and then we will extend to the unseparable case. The utility function is:

\[
U = \frac{C^{1-\sigma}}{1-\sigma} + \phi \frac{E^{1-\sigma}}{1-\sigma}, \quad \sigma \neq 1
\]  

(4)

where \( C \) denotes the consumption, \( \sigma \) is the intertemporal elasticity of substitution, \( \phi \) measures the relative contribution of public services to the utility.

The household is faced with the resource constraint:

\[
\dot{K}_p = (1-\tau)Y - C
\]  

(5)

where \( \tau \in (0,1) \), output \( Y \) can be used for government income, consumption and private capital investment. The household chooses consumption to maximize the discounted present value of utility.

2.3 Public services accumulation

In this paper, we consider the public services as a stock. The newly-accumulated public
services $\dot{E}$ depends on the direct government investment on public services $G_E$, and on the government expenditure on infrastructure $G_I$. Moreover, the quality of public services also affects the accumulation of public services. To be specific, increasing the public services will reduce the ability of the public services system to produce new services without increasing the government investment at the same time. We define the quality as $Q = \left(\frac{G_E}{E}\right)^{\eta}$, where $\eta \in (0,1)$, measures the degree of congestion. When $\eta \rightarrow 0$, there is no congestion effect. The accumulation function is:

$$\dot{E} = QG_E^{\mu_1}G_I^{\mu_2}E^{\tau - \mu_1 - \mu_2}\tag{6}$$

where $\mu_1, \mu_2 \in (0,1)$ and $\mu_1 + \mu_2 < 1$. Substituting $Q = \left(\frac{G_E}{E}\right)^{\eta}$ in equation (6), we can rewrite the accumulation function as:

$$\frac{\dot{E}}{E} = \left(\frac{G_E}{E}\right)^{\mu_1 + \eta} \left(\frac{G_I}{E}\right)^{\mu_2}\tag{7}$$

Thus, the stock of public services growth rate depends both on the government spending on public services and infrastructure per public services.

2.4 Government

The government provides infrastructure and public services. We assume the government collects proportional tax on output and can not issue any debt. To maintain a balanced budget, the government’s flow budget constraint is given by:

$$G_E + G_I = \tau Y\tag{8}$$

Assume that the public services investment and the expenditure on infrastructure hold a constant fraction of tax revenue, so that $G_I = v_I \tau Y$, $G_E = v_E \tau Y$. From equation (8), $v_E$ and $v_I$ must satisfy:

$$v_E + v_I = 1\tag{9}$$

3. The Decentralized Equilibrium
**Infrastructure, Public Services and Optimal Economic Growth**

Consider the optimal problem of the household. The household maximizes equation (3) subject to (5), taking the tax rate $\tau$, the share of infrastructure in the government spending $\nu_j$ and the stock of public services $E$ as given. The current-value Hamilton function can be written as:

$$H = \frac{C^{1-\sigma}}{1-\sigma} + \phi \frac{E^{1-\sigma}}{1-\sigma} + \lambda [(1-\tau)Y - C]$$  \hspace{1cm} (10)

where $\lambda$ is the shadow value of wealth.

In addition, the equilibrium requires that the following transversality condition be met:

$$\lim_{t \to +\infty} \lambda K_0 \exp(-\rho t) = 0$$  \hspace{1cm} (11)

which implies that in the long run, the private capital or the shadow value must converge to 0. In other words, the household have already made use of every chance.

From the F.O.C condition $\frac{\partial H}{\partial C} = 0$, we can get:

$$C^* = \frac{\lambda}{\beta}$$  \hspace{1cm} (12)

which means that under the optimal condition, the marginal utility of wealth equals the marginal utility of consumption.

The Euler equation is:

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau)(1-\alpha - \beta) \frac{Y}{K_p}$$  \hspace{1cm} (13)

Combining (2) (5) (12) (13) yields:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} [(1-\tau)(1-\alpha - \beta) \frac{Y}{K_p} - \rho]$$  \hspace{1cm} (14)

$$\frac{\dot{K}_p}{K_p} = (1-\tau) \frac{Y}{K_p} - \frac{C}{K_p} = (1-\tau) (G_s) \frac{Y}{K_p} - \frac{C}{K_p}$$  \hspace{1cm} (15)

Set $c = \frac{C}{K_p}$, denoting consumption-capital ratio and $e = \frac{E}{K_p}$, denoting public services-capital ratio, the equation system is obtained:

$$\frac{\dot{K}_p}{K_p} = (1-\tau)(\nu_j) \frac{Y}{K_p} - c$$  \hspace{1cm} (16)

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} [(1-\alpha - \beta) \frac{\dot{K}_p}{K_p} + e - \rho]$$  \hspace{1cm} (17)
\[ \frac{\dot{E}}{E} = (1 - v_j)^{\mu_j + \mu_j (1-\alpha)} v_j^{1-\alpha} e^{\frac{\mu_j + \mu_j (1-\alpha) \tau}{1-\alpha}} \]

\[ \frac{\dot{c}}{c} = \frac{1}{\sigma} [ (1 - \alpha - \beta) - 1 ] (1 - \tau) (v_j \tau)^{\alpha(1-\alpha)} e^{\beta(1-\alpha)} - \frac{\rho}{\sigma} + c \]

\[ \frac{\dot{e}}{e} = (1 - v_j)^{\mu_j + \mu_j (1-\alpha)} v_j^{1-\alpha} e^{\frac{\mu_j + \mu_j (1-\alpha) \beta}{1-\alpha}} \]

\[-(1 - \tau) (v_j \tau)^{\alpha(1-\alpha)} e^{\beta(1-\alpha)} + c \]

On the balanced-growth path, the output, the stock of private capital, the consumption, and the stock of public services all grow at the same rate \( \gamma \), so that \( \gamma = \frac{\dot{Y}}{Y} = \frac{K_p}{K_p} = \frac{\dot{C}}{C} = \frac{\dot{E}}{E} \), which makes the consumption-capital ratio and public services-capital ratio hold constant. Thus, when the economy on the balanced-growth equilibrium, \( \frac{\dot{c}}{c} = 0 \), \( \frac{\dot{e}}{e} = 0 \). Let \( \ddot{c} \), \( \ddot{e} \) denotes the steady-value of \( c \), \( e \), and set \( \dot{c} = 0 \) yields:

\[ \ddot{c} = \frac{\rho}{\sigma} - \frac{1}{\sigma} (1 - \alpha - \beta) - 1 \left[ (1 - \tau) (v_j \tau)^{\alpha(1-\alpha)} e^{\beta(1-\alpha)} \right] \]

Substituting (22) in (21) with \( \dot{c} = 0 \) yields an implicit function:

\[ Q(\ddot{c}, v_j, \tau) = 0 \]

Combining (16), (18), (19), (20) and (21) immediately yields the growth rate \( \gamma \) in the equivalent forms:

\[ \gamma_1 = \frac{\dot{K}_p}{K_p} = \frac{1}{\sigma} (1 - \alpha - \beta)(1 - \tau) (v_j \tau)^{\alpha(1-\alpha)} e^{\beta(1-\alpha)} - \frac{\rho}{\sigma} \]

\[ \gamma_2 = \frac{\dot{E}}{E} = (1 - v_j)^{\mu_j + \mu_j (1-\alpha)} v_j^{1-\alpha} e^{\frac{\mu_j + \mu_j (1-\alpha) \beta}{1-\alpha}} \]

\[ \gamma_1 = \gamma_2 = \gamma \]

The transversality condition implies the following constraint on the equilibrium:

\[ \gamma (1 - \sigma^{-1}) < \rho \]

which imposes constraint on the optimal fiscal policy.

To investigate the dynamics in the vicinity of the stationary state, the system (22)(23) can be linearized as:
Infrastructure, Public Services and Optimal Economic Growth

\[
\begin{bmatrix}
\dot{c} \\
\dot{\bar{c}}
\end{bmatrix} =
\begin{bmatrix}
\omega_{11} & \omega_{12} \\
\omega_{21} & \omega_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{c} - \bar{c} \\
\bar{e} - \bar{e}
\end{bmatrix}
\]

where:

\[
\omega_{11} = 1 > 0 ,
\]

\[
\omega_{12} = \frac{\beta}{1 - \alpha} \left[ \frac{1}{\sigma} (1 - \alpha - \beta) - 1 \right] (1 - \tau) (v_i \tau)^{\beta/(1 - \alpha)} - \frac{\beta}{(1 - \alpha)} > 0 ,
\]

\[
\omega_{21} = 1 > 0 ,
\]

\[
\omega_{22} = \frac{-(\mu_1 + \mu_2 + \eta)(1 - \alpha - \beta)}{1 - \alpha} (1 - v_j)^{\mu_1 + \eta} v_j^{\frac{\mu_1 + \mu_2 + \eta}{1 - \alpha} - (\mu_1 + \eta)} \frac{\mu_1 + \mu_2 + \eta}{1 - \alpha} \frac{(\mu_1 + \mu_2 + \eta)(\alpha + \beta - 1)}{1 - \alpha} < 0
\]

\[\Delta = \omega_{11} \omega_{22} - \omega_{12} \omega_{21} < 0\]

This condition always holds, given \(\sigma < 1 - \alpha - \beta\), which implies that the intertemporal elasticity of substitution should be small enough. Thus, in the vicinity of the balanced-growth equilibrium, the dynamics system is saddle path stable. As rational anticipators, the household will choose the initial value \((c_0, \bar{e}_0)\) on the balanced growth path, which can converge to the equilibrium. The initial value \((c_0, \bar{e}_0)\), equations (19) (20) (21) (22) and the transversality condition determine the economy’s balanced-growth path.

4. Optimal Government Expenditure Structure

In this section, we still consider the economy in the situation of decentralized equilibrium, so the “optimal” means maximizing the steady-state economy’s growth rate \(\gamma\).

4.1 The optimal tax rate

Assume there is a certain optimal tax rate \(\tau^*\). On one hand, when the tax rate \(\tau\) is lower than \(\tau^*\), the expenditure on infrastructure and the accumulation of public services are insufficient, which will lower the economy’s growth rate in the long run. On the other hand, when the tax rate \(\tau\) is higher than \(\tau^*\), the accumulation of private capital is limited and the growth rate will be
Research of Mathematical Economics No. 1 2008

lowered too. Therefore, it is straightforward that there is a trade-off between government spending and private investment.

Firstly, we take the composition of government expenditure as constant, implying \( d\gamma_1 = 0 \). The effect of an unexpected permanent tax rate change on the growth rate is:

\[
\frac{d\gamma_2}{dt} = (\mu_1 + \mu_2 + \eta)(1 - \nu_j)^{1/\eta} \nu_j^{\mu_1/\mu_2 + \eta} \left( \frac{\mu_1 + \mu_2 + \eta}{\eta(\mu_1 + \mu_2 + \eta)} \right) \left( \frac{1 - \alpha - \beta \bar{\varepsilon}}{1 - \alpha} - \frac{1}{1 - \alpha} \frac{\partial \bar{\varepsilon}}{\partial \tau} \bar{\varepsilon}^{1 - \alpha} \right)
\]

Consider the growth rate is maximized, so that \( \frac{d\gamma_2}{dt} = 0 \) and \( \frac{\tau}{1 - \alpha} - \frac{1 - \alpha - \beta}{1 - \alpha} \frac{\partial \bar{\varepsilon}}{\partial \tau} \bar{\varepsilon}^{1 - \alpha} = 0 \). Let

\[
\varepsilon_{\tau/\tau} = -\frac{\partial \bar{\varepsilon}}{\partial \tau} \frac{\tau}{\bar{\varepsilon}}
\]
denote the elasticity of public services-private capital ratio with respect to tax rate, \( \frac{\tau}{1 - \alpha} - \frac{1 - \alpha - \beta}{1 - \alpha} \frac{\partial \bar{\varepsilon}}{\partial \tau} \bar{\varepsilon}^{1 - \alpha} = 0 \) yields:

\[
\varepsilon_{\tau/\tau} = -\frac{1}{1 - \alpha - \beta}
\]

Similarly, we investigate the effect of tax rate change on growth rate \( \gamma_1 \):

\[
\frac{d\gamma_1}{dt} = \frac{1}{\sigma}(1 - \alpha - \beta) \nu_j^{\alpha(1 - \alpha)} \left( \frac{\alpha}{1 - \alpha} \frac{\tau}{1 - \tau} \right) \frac{1}{1 - \tau} + \frac{\beta}{1 - \alpha} \frac{\partial \bar{\varepsilon}}{\partial \tau}
\]

When the growth rate is maximized, \( \frac{\tau}{1 - \tau} = \frac{\alpha}{1 - \alpha} \frac{\beta e_{\tau/\tau}}{1 - \alpha} \). Substituting (25), the optimal tax rate is obtained:

\[
\tau^* = \alpha + \beta
\]

Thus, the optimal tax rate means that the share of government income in the whole output equals the government spending’s contribution to the whole output. If \( \beta \to 0 \), formula (27) generates the Barro’s tax-and-spending rule. In general case \( \beta > 0 \), \( \tau^* \) accounts for both direct and indirect effects of government spending on production. To be more specific, the sum total elasticity of output with respect to three factors, \( G_j, E \) and \( k_p \) equals \( 1(\alpha + \beta + (1 - \alpha - \beta) = 1) \), and the weights of private investment and government spending are \( (1 - \alpha - \beta) \) and \( \alpha + \beta \) respectively. The optimal tax rate measures the contribution of government spending to output precisely. In addition, formula (27) illustrates that the determination of optimal tax rate is independent of the composition of government expenditure and the technology of public services accumulation.
Infrastructure, Public Services and Optimal Economic Growth

4.2 The optimal composition of government expenditure

We can notice that the expenditure on infrastructure \( G_f \) and the investment on public services \( G_e \) affect the production in two different ways: the infrastructure is able to enhance production not only by indirect way of raising accumulation of public services, but also by raising output directly. The investment on public services can only promote production by increasing the stock of public services. By intuition, the optimal proportion of \( G_f \) to \( G_e \) is determined by their relative contribution to the production and public services accumulation.

Hold tax revenue neutral, so that \( d\tau = 0 \). The effect of the share of infrastructure expenditure change on the growth rate is:

\[
\frac{d\gamma_1}{d\nu_f} = \frac{1}{\sigma} (1 - \alpha - \beta)(1 - \tau) \nu_f^\alpha \nu_i^\beta \left( \frac{\alpha}{1 - \alpha} \nu_j^{-1} + \beta \frac{\partial \bar{e}}{\partial \nu_f} \bar{e}^{-1} \right)
\]

Set \( \varepsilon_{\nu_f} = -\frac{\partial \bar{e}}{\partial \nu_f} \), denoting the elasticity of public services-private capital ratio with respect to the fraction of infrastructure in the government spending. The optimal condition \( \frac{d\gamma_1}{d\nu_f} = 0 \) yields:

\[
\varepsilon_{\nu_f} = \frac{\alpha}{\beta}
\]

Similarly, the effect of the share of infrastructure expenditure change on the growth rate \( \gamma_2 \) is:

\[
\frac{d\gamma_2}{d\nu_f} = (1 - \nu_f) \frac{\mu_i + \mu_2 + \eta}{v_f} \frac{1}{1 - \alpha} \frac{\nu_f}{\nu_i^\alpha \nu_j^\beta \bar{e}^{-1}} \left( \frac{\mu_i + \mu_2 + \eta}{1 - \alpha} \nu_j^{-1} + \frac{\mu_1 + \mu_2 + \eta}{1 - \alpha} \frac{\partial \bar{e}}{\partial \nu_f} \bar{e}^{-1} \right)
\]

\( \chi = -(\mu_i + \eta)(1 - \nu_f)^{-1} + (\frac{\mu_i + \mu_2 + \eta}{1 - \alpha} - \mu_1 - \eta) \nu_j^{-1} + \frac{\mu_1 + \mu_2 + \eta(\alpha + \beta - 1)}{1 - \alpha} \frac{\partial \bar{e}}{\partial \nu_f} \bar{e}^{-1} \left( \frac{\mu_i + \mu_2 + \eta}{1 - \alpha} \right)
\]

With \( \frac{d\gamma_2}{d\nu_f} = 0 \), \( \chi = 0 \) yields:

\[
\frac{-(\mu_i + \eta)}{1 - \nu_f} = \frac{(\mu_i + \mu_2 + \eta)(\alpha + \beta - 1)}{1 - \alpha} \varepsilon_{\nu_f} = \frac{\mu_1 + \mu_2 + \eta}{1 - \alpha}
\]

Substituting with (28), the optimal share of \( G_f \) and \( G_e \) is obtained:

\[
\nu_f^* = \frac{\alpha(\mu_i + \mu_2 + \eta) + \beta \mu_2}{(\alpha + \beta)(\mu_i + \mu_2 + \eta)}
\]

\[
\nu_e^* = \frac{\beta(\mu_1 + \eta)}{(\alpha + \beta)(\mu_i + \mu_2 + \eta)}
\]
The formula (29), (30) may appear complicated at first sight, but the implication behind the results is straightforward. The first term $\alpha(\mu_1 + \mu_2 + \eta)$ in (29) could account for the direct effect of the infrastructure expenditure $G_i$ on production, and the second term $\beta\mu_2$ could account for the indirect effect on production through enhancing the accumulation of public services. The investment on public services $G_E$ has only indirect effect on production and its contribution is reflected by $\beta(\mu_1 + \eta)$. Increasing the fraction of government resources allocated to infrastructure has a positive effect on production and accumulation of public services, which raises the growth rate in the long run. Meanwhile, this increase reduces the scale of the investment on public services, which will bring down the accumulation of public services and the growth rate. The total effect of increasing the expenditure of infrastructure on growth rate is ambiguous, and there is a trade-off between the two kinds of government spending. Now, assuming $\eta = 0$, consider several particular cases.

1. If $\mu_1 \to 0$, then $v^*_I \to 1$ and $v^*_E \to 0$: because the investment on public services has no effect on the economy, increasing the expenditure of infrastructure has only positive effect.

2. If $\mu_2 \to 0$, then $\frac{v^*_I}{v^*_E} \to \frac{\alpha}{\beta}$: because now the expenditure on infrastructure has no indirect effect on production and public services are determined by the investment on public services, $\frac{\alpha}{\beta}$ accounts for their relative contribution to the production and the optimal composition is independent of the technology of public services accumulation.

3. If $\alpha \to 0$, then $\frac{v^*_I}{v^*_E} \to \frac{\mu_2}{\mu_1}$: because the expenditure on infrastructure has no direct effect on production, the optimal share is determined by their relative contribution to the accumulation of public services and independent of production technology.

4. If $\beta \to 0$, then $v^*_I \to 1$ and $v^*_E \to 0$: because the stock of public services has no effect on production and in turn the investment on public services has no effect, increasing the expenditure of infrastructure has only positive effect.

5. **The Central-planner Problem**
Infrastructure, Public Services and Optimal Economic Growth

The foregoing statement focuses on the decentralized economy, in which the household takes the stock of public services as constant and has no idea that their choice between consumption and capital investment may affect the stock of public services. It is possible that the household consumes too much and the accumulation of public services is insufficient.

If the economy is organized by a mercy central-planner, his optimal problem is:

$$\max V = \int_0^\infty \left( \frac{C^{1-\sigma}}{1-\sigma} + \phi \frac{E^{1-\sigma}}{1-\sigma} \right) \exp(-\rho t) \, dt$$

subject to:

$$\dot{K}_r = (1 - \tau) Y - C$$

The current-value Hamilton function can be written as:

$$L = \frac{C^{1-\sigma}}{1-\sigma} + \phi \frac{E^{1-\sigma}}{1-\sigma} + \theta_1[(1 - \tau) Y - C] + \theta_2 \left( \frac{G_E}{E} \right)^{\mu_1 + \eta} \left( \frac{G_L}{L} \right)^{\mu_2}$$

The following transversality conditions must hold:

$$\lim_{t \to \infty} \theta_1 \dot{K}_r \exp(-\rho t) = 0$$

$$\lim_{t \to \infty} \theta_2 \dot{E} \exp(-\rho t) = 0$$

Now, except for consumption and private capital, the central-planner also has to choose the stock of public services, the tax rate and the composition of government spending. According to

$$\frac{\partial L}{\partial \tau} = 0$$

and

$$\frac{\partial L}{\partial \nu_j} = 0$$

the optimal tax rate and the share of the expenditure on infrastructure, $\tau^*$ and $\nu_j^*$ satisfy the following equation:

$$\frac{\tau}{\nu_j} \left[ \frac{\mu_1 + \eta}{1 - \nu_j} + \frac{\mu_2 + \alpha (\mu_1 + \nu_j + \eta)(1 - \alpha)^{-1}}{\nu_j} \right] = \frac{(\mu_1 + \mu_2 + \eta)}{\alpha} \left[ \frac{\alpha}{(1 - \alpha)\tau} - \frac{1}{1 - \tau} \right]$$

Because the system is too complicated, we can not get the clean-cut results of $\tau^*$, $\nu_j^*$.

6. Multiplicatively Separable Utility Function

In section 2, we introduce the utility function that is separable in consumption and public services. Now we extend to the inseparable case. Consider the decentralized economy again, now
the optimal problem of household is:

$$\max \mathcal{V} = \int_0^\infty \left( \frac{C^\alpha E^{1-\alpha}}{1-\sigma} \right) \exp(-\rho t) dt \quad (37)$$

and the other conditions are the same with those in section 2 and 3. We can get the stationary growth rate as follows:

$$\gamma_1 = \frac{1}{\sigma} (1-\alpha-\beta)(1-\tau) \left( \frac{\nu \tau}{\sigma} \right)^{\gamma_1} \left( \frac{\rho}{\sigma} \right)$$

$$\gamma_2 = (1-\nu) \frac{\mu + \eta}{\tau} \frac{1}{\alpha} \left( \frac{\rho}{\sigma} \right)$$

$$\gamma_1 = \gamma_2 = \gamma$$

which imply that additively and multiplicatively separable utility functions obtain the same growth rate and optimal fiscal policy. Thus, it is the technology of production and public services accumulation that affect the optimal fiscal policy and the determination of optimal fiscal policy is independent of preference structure.

7. Conclusion

This paper studies the optimal tax rate and the allocation of government resources between infrastructure and public services to maximize the economy’s growth rate. In decentralized equilibrium, the optimal proportion of government spending to the output should equal to its contribution to the output, and this result is similar to the Barro’s rule. Because we divide government spending into two parts and they affect the output in different ways, there is a trade-off. The optimal fraction of the two kinds of government expenditure depend only on the technology of production and accumulation of public services.

Secondly, we examine the first-best optimum, when the government as a central-planner is to maximize the household welfare. Unfortunately, the explicit results cannot be obtained. Moreover, we compare the separable utility function with the inseparable utility function. The results show that the preference structure doesn’t affect the optimal fiscal policy. It is the production and public services accumulation technology that determine the government expenditure structure.

Finally, this model is not perfect and could be extended in several ways. For example, this paper doesn’t discuss the effect of temporary change in tax rate and the composition of
Infrastructure, Public Services and Optimal Economic Growth

government spending on household welfare. The congestion cost could be discussed further, too. These compliment will be worthwhile to the present analysis.

References


Arrow, M. Kurz, “Public Investment, the Rate of Return, and Optimal Fiscal Policy”, Johan Hopking Press, 1970.


(Responsible Editor: Liu jingrong) (Proofreader: Guo Yumei)
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

Huo Zhen
(School of Economics, Renmin University of China)

Abstract: This paper studies the relationship between fiscal decentralization and economic growth in China. We distinguish three kinds of expenditure decentralization—physical capital, human capital and government administration expenditure decentralization and use a panel data set of 28 provinces over 1995-2003 to investigate them whether enhance economic growth. We find that government administration expenditure decentralization has a positive effect on China’s economic growth, but there is no relationship between capital expenditure decentralization and economic growth.

Keywords: fiscal decentralization, public expenditure composition, economic growth

1. Introduction

Since 1980s, fiscal decentralization has been a useful tool for many countries in the world to optimize their fiscal system and accelerate economic growth. On one hand, 84% of 75 economies which underwent economic transition with a population of more than 5 million had chosen to devolute fiscal power from the national government to the subnational government. On the other hand, some traditional countries with federal systems began to reconsider their decentralized fiscal system. The market-orientated economy reform embarked on in 1978 in China, at the same time, the centralized fiscal system was also reformed greatly to improve efficiency in public sector, for instance, the “baoganzhi” reform and the “fenshuizhi” reform. The general trend of the fiscal reform is that the central government transfers power to the local government. With China’s economy growing at a considerable high rate, a question raised: the fiscal decentralization in China whether enhanced or hindered economic growth?
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

The relationship between fiscal decentralization and economic growth has received much debate in the analytical literature. One strand in favor of fiscal decentralization postulates that the costs of providing public services vary with regions. Local government knows local residents’ preference and has information advantages when delivering public good (Oates, 1972). Tiebout’s (1956) “vote by foot” model emphasizes the mobility of factors and competition among local governments will ensure economic efficiency. The other strand in favor of fiscal decentralization focuses on the yardstick competition and the restraint mechanism which forces the local government performs well. However, there are a lot of researches questioning fiscal decentralization: first, the central government has advantage of economies of scale, so competition among local governments may bring about externality costs. If the degree of fiscal decentralization exceeds a certain level, it may lower the economic efficiency. Secondly, fiscal decentralization requires the local autonomy, which calls for democratic politics. These social conditions can not be satisfied in developing and transitional countries usually.

As concerns the empirical research, the results are also ambiguous. Xie, Zou and Davoodi(1999) evaluated the effect of fiscal decentralization in the United States and found that there was a negative relationship between fiscal decentralization and economic growth. Davoodi and Zou(1998) found a negative relationship between fiscal decentralization and economic growth in developing countries, but none in developed countries. Akai and Sakata(2002) reached the opposite conclusion that fiscal decentralization has a positive effect.

Studies on China’s fiscal decentralization do not get the same conclusion, either. Zhang and Zou(1998) first tested the impacts of fiscal decentralization on China’s economic growth, and they found that fiscal decentralization was detrimental to economic growth. Lin and Liu(2000) treated fiscal decentralization as a part of China’s economic reform, considered some variables changed in the reform and reached the opposite conclusion. Zhang and Gong(2005) carefully investigated the effect of fiscal decentralization by considering the economic development level, geographic location. They found that fiscal decentralization had a negative effect before 1994 and a positive effect after 1994.

The empirical analysis reaching different conclusion is mainly due to scholars choosing different ways to measure the degree of fiscal decentralization. The proxy for fiscal decentralization is the most important factor to determine the estimation result. Though studies on fiscal decentralization in China are numerous, few consider the effect of expenditure composition.
on the relationship between fiscal decentralization and economic growth. One exception is that Jia, Guo and Liu(2006) found that capita expenditure decentralization level is not growth-maximizing. In fact, previous studies do not distinguish what local government buys: spending on capital accumulation or wages and salaries may result in different economic outcomes. When local government gets fiscal power from central government, they will determine which item to spend on, which could lead to higher or lower economic growth rate.

The innovation of this paper is that we distinguish three kinds of expenditure decentralization: physical capital, human capital and government administration expenditure decentralization. This paper will focus on evaluating the effect of the three kinds of decentralization on China’s economic growth. The rest of the paper is organized as follows. Section 2 introduces a basic analytical model. Section 3 discusses the measure of fiscal decentralization, sets out the estimation equation and gives a brief introduction of the data set. Section 4 reports the estimation results. Section 5 concludes and points to some limitation of this study.

2. Analytical Framework

Follow Davoodi and Zou(1998)’s framework. Consider a closed economy with a perfect competitive market. There is an infinitively-lived representative household who produces a single good, which can be consumed or invested. The public expenditure is carried out by two levels of governments: central government and local government. They levy an income tax on output to maintain a balance budget and spend the government income according to a fixed proportion. At the same time, the two levels of governments provide public goods for the production.

Assume the production adopt a Cobb-Douglas technology:

\[ Y = G^a_j G^\beta_j K^\alpha - \beta_p \]  \hspace{1cm} (2.1)

where \( G_j \) denotes the central government expenditure, \( G_j \) denotes local government expenditure, \( K_p \) denotes the stock of private capital. We also assume \( \alpha, \beta \in (0,1) \) and \( \alpha + \beta < 1 \), which are the elasticity of output with respect to the three factors.

The rational infinitively-lived household’s object is to maximize the discount stream of utility:

\[ \max_{c} V = \int_{0}^{\infty} \frac{c^{1-s}}{1-\sigma} e^{-\gamma t} dt \]  \hspace{1cm} (2.2)
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

Where $C$ denotes the consumption, $\sigma$ is the intertemporal elasticity of substitution, $\rho > 0$, is the exogenous discounted rate.

The household is faced with the resource constraint:

$$\dot{K}_p = (1 - \tau)Y - C$$  \hspace{1cm} (2.3)

Where $\tau \in (0,1)$ denotes the tax rate.

Assume $G_j = \nu_j \tau Y$, $G_s = \nu_s \tau Y$, which means the central government and the local government hold a constant fraction of tax revenue. $\nu_j$ and $\nu_s$ measure the degree of fiscal decentralization, they satisfy the equation:

$$\nu_j + \nu_s = 1$$  \hspace{1cm} (2.4)

The current-value Hamiltonian function and the transversality condition can be given as:

$$H = \frac{C^{\sigma}}{1 - \sigma} + \lambda[(1 - \tau)Y - C]$$  \hspace{1cm} (2.5)

$$\lim_{t \to \infty} \lambda K_p \exp(-\rho t) = 0$$  \hspace{1cm} (2.6)

From the F.O.C condition and the Euler equation, the optimal path of consumption is obtained:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}[(1 - \tau)(1 - \alpha - \beta) \frac{Y}{K_p} - \rho]$$  \hspace{1cm} (2.7)

By setting $c = \frac{C}{K_p}$, which denotes consumption-capital ratio, the dynamic system can be given as:

$$\frac{\dot{K}_p}{K_p} = (1 - \tau) \frac{Y}{K_p} - c$$  \hspace{1cm} (2.8)

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}[(1 - \alpha - \beta) - 1](1 - \tau) \frac{Y}{K_p} - \frac{\rho}{\sigma} + c$$  \hspace{1cm} (2.9)

On the balanced growth path, the output, the stock of private capital and the consumption all grow at the same rate $\gamma$:

$$\gamma = \frac{\dot{Y}}{Y} = \frac{\dot{K}_p}{K_p} = \frac{\dot{c}}{c}$$  \hspace{1cm} (2.10)

Thus, when the economy on the balanced-growth equilibrium $\frac{\dot{c}}{c} = 0$, which can yield the growth rate $\gamma$:
\begin{equation}
\gamma = \frac{1}{\sigma}[\!(1-\alpha-\beta)(1-\tau)\!r^{(\alpha+\beta)/(1-\alpha-\beta)}v^a_{\beta}(1-\alpha-\beta) - \rho]\!
\end{equation}

At the same time, the growth-maximizing degree of fiscal decentralization is:

\begin{equation}
\nu^*_f = \alpha \quad \nu^*_s = \beta
\end{equation}

Therefore, the degree of fiscal decentralization will influence the economic growth rate. It may improve or worsen the economic efficiency.

3. Empirical Analysis

3.1 Measure of fiscal decentralization

The key point of the studying of fiscal decentralization lies on the measure of fiscal decentralization. The previous research usually used the ratio of local government income (expenditure) to the central government income (expenditure). The proxy for fiscal decentralization used in previous literature can be summarized as follows:

1. Zhang and Zou(1998) designed 6 proxies for fiscal decentralization according to government expenditure:
   - DC1= total provincial government spending / total central government spending
   - DC2= per-capita provincial government spending / per-capita central government spending
   - DC3= provincial budgetary government spending / central budgetary government spending.
   - DC4=provincial extra-budgetary government spending/central extra-budgetary government spending.
   - DC5=provincial per-capita budgetary government spending/central per-capita budgetary government spending.
   - DC6=(provincial extra-budgetary government spending / provincial income) / (central extra-budgetary government spending/national income)

The ratio of local spending to central spending can reflex the local government’s autonomy in expenditure. However, Liu and Liu(2000) pointed out that this measure was questionable because the degree of fiscal decentralization was totally determined by the local spending. The larger the local spending is, the greater the degree of fiscal decentralization is. This situation is not accord with the reality.

2. Lin and Liu(2000) measured the degree of fiscal decentralization by the marginal retention
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

rate of locally collected budgetary revenues by provincial governments and they divided the provinces into 4 types according to central-local fiscal arrangement. Though this measure seems to be good, Zhang and Gong(2005) stated that it made some provinces get the same degree of fiscal decentralization while their fiscal power differs greatly.

3. Zhang and Gong(2005) considered the effect of transfer between central government and local government. They designed 4 types of measure of fiscal decentralization:

DC1= provincial budgetary government income / central budgetary government income
DC2=provincial budgetary government spending / central budgetary government spending
DC3=(provincial budgetary government spending—transfer) / central budgetary government spending.
DC4= total provincial government spending / total central government spending

Zhang and Zou(1998), Davoodi and Zou(1998) emphasized that the composition of government expenditure affected the relationship between fiscal decentralization and economic growth. By intuition, capital and infrastructure spending have positive growth effects while administration and wages spending have negative growth effects. Most of the scholars overlooked the composition of government expenditure when studying fiscal decentralization. Though Zhang and Zou(1998) considered this point, they just treated the composition as an independent variable. In this paper, we design 3 types of proxy for fiscal decentralization and want to evaluate each of them whether improves economic growth or not:

FD1= provincial budgetary government physical capital spending / central budgetary government physical capital spending
FD2=provincial budgetary government human capital spending / central budgetary government human capital spending
FD3=provincial budgetary government administration spending / central budgetary government administration spending.

3.2 Econometric specification and data sources

We use a lin-log model and the growth regression will be estimated on a cross-province panel data using the ordinary least squares technique:

$$PGD_{n} = \beta_{0} + \beta_{1} \log(FGD_{n}) + \beta_{2} \log(FD_{n}) + \beta_{3} \log(M_{n}) + u_{t} + \lambda_{t} + \varepsilon_{nt}$$  (3.1)
Research of Mathematical Economics No. 1 2008

where \( i = 1, \ldots, I \) and \( t = 1, \ldots, T \) refer to province \( i \) at time \( t \). \( PGDP \) denotes per-capita GDP, \( FGDP \) denotes the ratio of local government income to the provincial GDP, \( FD \) denotes the measure of fiscal decentralization, \( \alpha \) is a vector of control variables, \( u_t \) denotes the province fixed-effect and \( \lambda_t \) denotes the time fixed-effect, \( \epsilon_{it} \) is the disturbance term. The Hauseman specification test rejects the random-effect model, so equation (3.1) is a two-way fixed-effect model. According to expectation, the coefficient of \( FD1 \) and \( FD2 \) is positive and significant and the coefficient of \( FD3 \) is negative and significant.

Our empirical analysis is based on the province-level panel data from 29 provinces for the period 1995-2003. Because in 1993 and 1994 the fiscal system conducted “fenshuizhi” reform, we choose the sample from 1995 to ensure the stationary of the data. \( FGDP \) could account for the general tax rate. The vector \( \alpha \) consists of a set of variables: (1) labor, the number of employed people; (2) I account for the investment measured by the total investment in fixed assets; (3) FDI, the foreign direct investment. \( FD1 \), physical capital spending including expenditure for capital construction, enterprises innovation funds, science and technology innovation funds, expenditure for supporting rural production. \( FD2 \), human capital spending including operating expenses for health, science, education, culture, sports and broadcasts. \( FD3 \), government administration spending including expenditure for government administration, diplomacy, public security agency, procuratorial agency and court of justice. All the data are taken from China statistical yearbook and China fiscal yearbook.

4. Regression Results

We estimate regression(3.1) using(1) the full sample---including 29 provinces from 1995-2003; (2) three year average data, because the fiscal decentralization effect may not be able to explain the current fluctuation economic growth. We test the two kinds of data with and without
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

control variables. The baseline regression includes the following regressors: the constant, $FGDP$---the ratio of local government income to the provincial GDP, $FD$---fiscal decentralization, $u_t$ and $\lambda_t$---province and time fixed-effect. As for the three year average data, we also consider the linear regression model:

$$PGDP_\mu = \beta_0 + \beta_1 FGDP_{\mu} + \beta_2 FD_{\mu} + \beta_3 M_{\mu} + u_t + \lambda_t + \epsilon_\mu$$  \hspace{1cm} (4.1)

The main results are summarized in 9 tables. The $t$-statistics are in parentheses in the tables (in Appendix).

Tables 7-9 show that there is a positive relationship between government administration expenditure decentralization and economic growth for both full sample and three year average sample data and for both (3.1) regression equation and (4.1) regression equation. The point estimate is statistically significant at the 10% level. The $t$-ratio are 6.6 (Table 7; column 4), 3.8 (Table 8; column 4), and 1.76 (Table 9; column 4). To examine whether the estimated effect of $FD3$ is sensitive to inclusion of other variables, we add control variables orderly. The estimates hardly vary.

Tables 1-6 show that there are none significant relationship between capital (physical and human) expenditure decentralization and economic growth. All the point estimates are not statistically significant at the 10% level except the estimates in Table 1; column 3 and Table 1; column 4. Whether adding control variables or not effect the basic result little.

As for other explanatory variables in the regressions, $FGDP$ ---the ratio of local government income to the provincial GDP is positively related to growth and significant in statistic. Labor--- the number of employed people detrimental to the economic growth. This is because we use the per-capita GDP as the dependent variable. FDI---the foreign direct investment has positive effect on the economic growth and this is accord with our expectations.

To sum up, the regression results show that: (1) There is positive relationship between government expenditure fiscal decentralization and economic growth. (2) There is no relationship between physical and human capital expenditure fiscal decentralization and economic growth. The results are opposite to our expectation, we could only provide some possible reasons: (1) When the local government get the fiscal power, they may try to maximize their budget for wages, salaries and other items which can improve the officials’ welfare, which may stimulates consumption and rise GDP temporarily. (2) The local government has no incentive to improve human capital
condition, because it is difficult to evaluate whether the human capital condition get better or not.

(3) The competition among local government officials to raise GDP will force them to invest on infrastructure, but it may result in investing too much and being detriment of the economy.

5. Conclusion

In this paper we have discussed the relationship between three kinds of fiscal decentralization and economic growth. First, we set up a simple analytical model to give a basic result of fiscal decentralization and economic growth. Secondly, we design three new proxies for fiscal decentralization considering the composition of government expenditure. Then we estimated the effect of three kinds of fiscal decentralization with a two-way fixed-effect model. From our sample, we find that there is no relationship between capital expenditure decentralization and economic growth in China in the period 1995-2003. However, government administration expenditure decentralization has a positive effect on economic growth. We give our explanations for the results.

Finally, the current analysis is not perfect and could be extended in several ways. For example, the analytical model could be made more specific to include both composition of government expenditure and fiscal decentralization; the regression equation could use dynamic panel data model to test if the fiscal decentralization has a lag-effect. Due to the data and our estimation technology, the results could be modified with larger sample and better regression technique.
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

Appendix

Table 1

<table>
<thead>
<tr>
<th></th>
<th>Dep. Var: Per Capita GDP</th>
<th>Full sample data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Indep. Var</strong></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.08*</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>(2.57)</td>
<td>(0.45)</td>
</tr>
<tr>
<td>log(FGDP)</td>
<td>0.77*</td>
<td>0.8*</td>
</tr>
<tr>
<td></td>
<td>(5.21)</td>
<td>(5.32)</td>
</tr>
<tr>
<td>log(FDI)</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(-0.45)</td>
<td>(-0.3)</td>
</tr>
<tr>
<td>log(Labor)</td>
<td>-0.25*</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(-1.14)</td>
<td>(-0.39)</td>
</tr>
<tr>
<td>log(l)</td>
<td>-0.73*</td>
<td>-0.78*</td>
</tr>
<tr>
<td></td>
<td>(-7.68)</td>
<td>(-8.04)</td>
</tr>
<tr>
<td>log(FDI)</td>
<td></td>
<td>0.05*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.15)</td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th></th>
<th>Dep. Var: Per Capita GDP</th>
<th>Three year average data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Indep. Var</strong></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.43</td>
<td>5.66*</td>
</tr>
<tr>
<td></td>
<td>(-0.64)</td>
<td>(2.53)</td>
</tr>
<tr>
<td>log(FGDP)</td>
<td>0.38*</td>
<td>1.06*</td>
</tr>
<tr>
<td></td>
<td>(2.11)</td>
<td>(3.61)</td>
</tr>
<tr>
<td>log(FDI)</td>
<td>-0.13</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(-0.94)</td>
<td>(-0.64)</td>
</tr>
<tr>
<td>log(Labor)</td>
<td>-1.02*</td>
<td>-0.44</td>
</tr>
<tr>
<td></td>
<td>(-2.84)</td>
<td>(-1.3)</td>
</tr>
<tr>
<td>log(l)</td>
<td></td>
<td>-0.8*</td>
</tr>
</tbody>
</table>
Research of Mathematical Economics No. 1 2008

<table>
<thead>
<tr>
<th></th>
<th>(4.44)</th>
<th>(4.68)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(FDI)</td>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(1.37)</td>
<td></td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 3

Dep. Var: Per Capita GDP Three year average data

<table>
<thead>
<tr>
<th>Indep.Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.16</td>
<td>0.15</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(-0.76)</td>
<td>(0.55)</td>
<td>(0.57)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>FGDP</td>
<td>0.09*</td>
<td>0.11*</td>
<td>0.11*</td>
<td>0.11*</td>
</tr>
<tr>
<td></td>
<td>(4.5 )</td>
<td>(4.98)</td>
<td>(4.44)</td>
<td>(4.41)</td>
</tr>
<tr>
<td>FD1</td>
<td>-0.31</td>
<td>-0.39</td>
<td>-0.49</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>(-0.19)</td>
<td>(-0.24)</td>
<td>(-0.28)</td>
<td>(-0.45)</td>
</tr>
<tr>
<td>Labor</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.89)</td>
<td>(-1.87)</td>
<td>(-1.66)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>-0.01</td>
<td>-0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.15)</td>
<td>(-0.68)</td>
<td></td>
</tr>
<tr>
<td>FDI</td>
<td></td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.92)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.93</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Table 4

Dep. Var: Per Capita GDP Full sample data

<table>
<thead>
<tr>
<th>Indep.Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.93*</td>
<td>1.31</td>
<td>4.2*</td>
<td>3.89*</td>
</tr>
<tr>
<td></td>
<td>(-2.25)</td>
<td>(0.75)</td>
<td>(2.57)</td>
<td>(2.4 )</td>
</tr>
<tr>
<td>log(FGDP)</td>
<td>0.76*</td>
<td>0.78*</td>
<td>0.68*</td>
<td>0.66*</td>
</tr>
<tr>
<td></td>
<td>(5.06)</td>
<td>(5.18)</td>
<td>(4.98)</td>
<td>(4.92)</td>
</tr>
<tr>
<td>log(FD2)</td>
<td>0.02</td>
<td>0.07</td>
<td>0.11</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.6 )</td>
<td>(1.05)</td>
<td>(0.98)</td>
</tr>
<tr>
<td>log(Labor)</td>
<td>-0.3</td>
<td>-0.11</td>
<td>-0.09</td>
<td></td>
</tr>
</tbody>
</table>
### Table 5

<table>
<thead>
<tr>
<th>Indep. Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.05</td>
<td>6.68*</td>
<td>6.89*</td>
<td>6.17*</td>
</tr>
<tr>
<td></td>
<td>(-0.06)</td>
<td>(2.77)</td>
<td>(3.28)</td>
<td>(2.92)</td>
</tr>
<tr>
<td>log(FGDP)</td>
<td>0.33*</td>
<td>1.02*</td>
<td>1.07*</td>
<td>1.06*</td>
</tr>
<tr>
<td></td>
<td>(1.72)</td>
<td>(3.46)</td>
<td>(4.18)</td>
<td>(4.18)</td>
</tr>
<tr>
<td>log(FD2)</td>
<td>-0.07</td>
<td>0.16</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(-0.17)</td>
<td>(0.44)</td>
<td>(0.58)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>log(Labor)</td>
<td>-1.08*</td>
<td>-0.51</td>
<td>-0.44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.96)</td>
<td>(-1.47)</td>
<td>(-1.27)</td>
<td></td>
</tr>
<tr>
<td>log(l)</td>
<td></td>
<td></td>
<td></td>
<td>-0.79*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-4.58)</td>
</tr>
<tr>
<td>log(FDI)</td>
<td></td>
<td></td>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.63)</td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.91</td>
<td>0.92</td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Indep. Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.04</td>
<td>0.18</td>
<td>0.19</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.63)</td>
<td>(0.65)</td>
<td>(0.57)</td>
</tr>
<tr>
<td>FGDP</td>
<td>0.10*</td>
<td>0.11*</td>
<td>0.11*</td>
<td>4.79*</td>
</tr>
<tr>
<td></td>
<td>(5.03)</td>
<td>(5.31)</td>
<td>(4.94)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>FD2</td>
<td>-0.95</td>
<td>-0.35</td>
<td>-0.4</td>
<td>-0.48</td>
</tr>
</tbody>
</table>
### Table 7

<table>
<thead>
<tr>
<th>Indep.Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.31*</td>
<td>5.5*</td>
<td>8.17*</td>
<td>7.88*</td>
</tr>
<tr>
<td></td>
<td>(3.41)</td>
<td>(3.29)</td>
<td>(5.34)</td>
<td>(5.21)</td>
</tr>
<tr>
<td>log(FGDP)</td>
<td>0.4*</td>
<td>0.43*</td>
<td>0.34*</td>
<td>0.32*</td>
</tr>
<tr>
<td></td>
<td>(2.64)</td>
<td>(2.83)</td>
<td>(2.49)</td>
<td>(2.43)</td>
</tr>
<tr>
<td>log(FD3)</td>
<td>0.71*</td>
<td>0.75*</td>
<td>0.76*</td>
<td>0.75*</td>
</tr>
<tr>
<td></td>
<td>(5.53)</td>
<td>(5.81)</td>
<td>(6.59)</td>
<td>(6.6)</td>
</tr>
<tr>
<td>log(Labor)</td>
<td>-0.42*</td>
<td>-0.2</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.08)</td>
<td>(-1.12)</td>
<td>(-1.06)</td>
<td></td>
</tr>
<tr>
<td>log(I)</td>
<td>-0.6*</td>
<td>-0.68*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-7.62)</td>
<td>(-8.16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(FDI)</td>
<td></td>
<td>0.05*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.63)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
</tr>
</tbody>
</table>

### Table 8

<table>
<thead>
<tr>
<th>Indep.Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>6.17*</td>
<td>9.85*</td>
<td>9.7*</td>
<td>8.98*</td>
</tr>
<tr>
<td></td>
<td>(4.05)</td>
<td>(4.58)</td>
<td>(5.27)</td>
<td>(4.78)</td>
</tr>
<tr>
<td>log(FGDP)</td>
<td>-0.23</td>
<td>0.37</td>
<td>0.47*</td>
<td>0.48*</td>
</tr>
</tbody>
</table>

---

*Research of Mathematical Economics No. 1 2008*
Fiscal Decentralization, Public Expenditure Composition and Economic Growth in China

<table>
<thead>
<tr>
<th></th>
<th>(-1.14)</th>
<th>(1.17)</th>
<th>(1.73)</th>
<th>(1.77)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(FD3)</td>
<td>1.41*</td>
<td>1.23*</td>
<td>1.14*</td>
<td>1.1*</td>
</tr>
<tr>
<td></td>
<td>(4.1)</td>
<td>(3.61)</td>
<td>(3.89)</td>
<td>(3.8)</td>
</tr>
<tr>
<td>log(Labor)</td>
<td>-0.77*</td>
<td>-0.25</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.34)</td>
<td>(-0.82)</td>
<td>(-0.63)</td>
<td></td>
</tr>
<tr>
<td>log(I)</td>
<td>-0.64*</td>
<td>-0.73*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-4.48)</td>
<td>(-4.78)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(FDI)</td>
<td></td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.93</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 9

<table>
<thead>
<tr>
<th>Dep. Var: Per Capita GDP</th>
<th>Three year average data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indep. Var</td>
<td>(1)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>(-1.26)</td>
</tr>
<tr>
<td>FGDP</td>
<td>0.07*</td>
</tr>
<tr>
<td></td>
<td>(3.15)</td>
</tr>
<tr>
<td>FD3</td>
<td>7.71*</td>
</tr>
<tr>
<td></td>
<td>(1.25)</td>
</tr>
<tr>
<td>Labor</td>
<td>-0.02*</td>
</tr>
<tr>
<td></td>
<td>(-2.13)</td>
</tr>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>FDI</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Ad R-square</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Note: *$t$* statistics are in parentheses.

**"***means the coefficients are significant at 5% level.

References

Research of Mathematical Economics No. 1 2008


(Responsible Editor: Shen Jing) (Proofreader: Lu Hong)
Labor Market Distortion and Price Adjustment:
Static Analysis on Chinese Economy
within a General Equilibrium Framework

Qiu Zhesheng
(School of Economics, Renmin University of China)

Abstract: This paper estimates the optimal labor allocation and the corresponding relative price by constructing a close general equilibrium model, concluding that the distortion of labor allocation in China is quite significant. The model adopted in this paper is based on CGE model, but has been simplified for multiply reasons, which guarantees the relative independence on such particular statistical data as the Input-output tables and the statistical yearbooks. As a result, simulating and analyzing the yearly equilibrium requires nothing more than 12 necessary and 4 additional Indicators. According to the simulation based on the data over 1991–2007, the long-run tendency of the labor market distortion and the corresponding price adjustment during last 17 years is illustrated in several graphs. Finally, this paper combines the outcomes of the simulation with some other indicators derived from the raw data, and divides the last 17 years into 3 sequential developing stages.

Keywords: Urban-Rural Disparity, Labor Market Distortion, Price Adjustment, General Equilibrium

1. Introduction

The nominal urban-rural income disparity in China had gone up to be 3.3:1 in 2007 after sustaining increase for 23 years1, which is extremely high by international standard. In fact, this disparity is underestimated rather than aggrandized because of the neglect of gray income and institutional discrimination. Tianyu Yang (2007) has estimated the gray income of different groups of people, and concludes that the actual income of richer people in China is much higher than what economic indicators have presented. Qiu (2008) Computes the actual Gini Coefficient based on the estimation of Yang, and finds that it is at least 0.60, rather than 0.47. According to the prevailing

---------------------------------------------------------------
1 According to statistical yearbooks from the National Bureau of Statistics of China
estimation, the actual income disparity between urban and rural areas is no less than 5:12.

W. Arthur Lewis (1954) constructed the Dual Sector model and divided the economy into traditional sectors and modern sectors. Ranis and Fei (1961) gave micro-foundations to the Lewis model, providing a link back to neoclassical growth theory. In the Lewis-Ranis-Fei model, urban-rural inequality in developing countries is due to the process of industrialization. However, in most former centre planning countries, institution also plays a significant role in making the disparity even larger.

China’s dual economy began with the forging ahead strategy. In the 1950’s when the PRC was newly established, the centre government implemented the long-term policy of priority development of heavy industry, and were in need of a great amount of capital. However, the most serious problem the centre government encountered then is the scarce of labor, capital, and technology. In order to accomplish the primary capital accumulation as soon as possible, the centre government had to extract surplus value from the agriculture by “Price Scissors” to support industrialization, which is known as the “policy of broad sense benevolence”.

Such Imbalanced developing strategy results in large income differential between urban and rural households. However, permanent migration in China has been limited by the system of official registration, whereby households must have a “Hukou” in order to legally reside in an urban area. Without this registration, access to urban amenities such as housing and education, is limited and quite expensive. (Fan Zhai 2004) As a result, the course of urbanization fell far behind that of industrialization.

The inner mechanism of this process can be interpreted in brief. Imbalanced growth rate and sticky relative price of agricultural products make agricultural revenue much less than that of other sectors. However, labor market barriers limit the migration of labor. Then, part of the people who intended to switch to non-agricultural sectors had to turn to informal sectors in urban areas or back to agriculture. Nowadays, about 940 million people are restricted by agriculture Hukou, which consists about 70% of the total population. Just like what Geertz (1936) presented in his theory of involution, the density of Chinese farm labor is too high.

Supposing that labor is homogenous, labor in the less productive sectors tends to transfer to more productive sectors. Then the quantity of products in those less productive sectors would

\(^2\) Some claims that it is 6:1.

\(^3\) Firstly proposed by Mao Zedong in the middle of 1950s
**Labor Market Distortion and Price Adjustment**

decrease while other sectors are inversed. Productivity of different sectors tends to converge, due to the diminishing marginal productivity of labor. Relative price will adjust to fit the new output level automatically. If the transfer of labor is not free, then the convergence is not easy to realize. In order to analyze this mechanism, a model based on general equilibrium theory is to be constructed.

CGE (computable general equilibrium) model is widely used since the late 1980s, and introduced into China in the early 1990s. However, the ordinary CGE model consists of more than 20 equations and requires data in many sectors. Additionally, the model is based on a series of assumptions and the intertemporal impacts are neglected, which is even inconsistent. (Yi Gong 2007) The results differ when the closure methods differ, thus more precise statistical data does not necessarily mean better outcomes.

Recent research of China’s CGE models are mainly in the World Bank (Fan Zhai), and development research center of the state council (Shantong Li). Some paper for master degree salso developed China’s CGE models (Juan Sheng 2005, and Yingqi Ma 2007). Most work only analyzes some particular years or some particular sectors; not much work has been done on the long-run tendency of China’s dual economy. In this paper, I will develop a simplified model with 12 endogenous variables and simulate the equilibrium condition over 1991~2007. Because of the simplicity of the model, it is possible to the guarantee the integrity of sequential data, and then some of the parameters in the production function and utility function can be estimated by linear regression models instead of being arbitrarily set at the very beginning.

In this model, labor allocation is the core, while capital allocation is not free. It does not mean that allocation of capital is not important. As the mechanism of interest rate marketization has not been well constructed, this paper merely deals with the labor allocation and price of real goods. This compromise makes this model not convincing enough to give the optimal recourses allocation of the real economy system, while the observation on the long-run trend is still useful.

2. **The General Equilibrium Model**

2.1 On the Producers

Cobb-Douglas production function fits modern sectors well, while China’s agriculture has its own properties. Capital can be accumulated, while land is limited. The maximal land productivity
of agriculture is determined by the infrastructure, technology and local climates; labor is not so important in China. When the density of labor on land is too high, the elasticity of labor-output is approaching 0, which means that the labor is satiated. This kind of system errors make the outcome of numerical simulation of surplus labor less than what it should be.

Another problem is about the capital stock. Now the most prevailing way to evaluate the capital stock is to choose one year as the base year, and then take the investment and depreciation into account (the method of everlasting duration). In this model, elasticity of labor-output is firstly estimated, then, factors except labor were calculated. The model considers other factors as a whole, because the main target is to calculate the distortion rate of the labor market, while capital is excluded technically.

Assume that return to scale is constant in each sector, and labor income equals marginal productivity of labor. It doesn’t matter whether it is the fact in the real economy. The later assumption only provides a baseline for quantifying the labor transferring. In this model, the elasticity of each factor does not mean the share of income, because capital market is not considered here. As a result, this model chooses the Income-expenditure balance as the closure condition, and imposed the fixed structure of expenditure for agents in one single year.

2.2 On the Consumers

In this model, demand function is used to adjust price with the changing output, variables of price and quantity of real consumption is included in the function. The function abstracts consumer’s behavior in the long run. As for China, the most important property which ought to be included in the function is the decreasing Engel Coefficients over time. The function cannot be too complex for parameter estimation.

Obviously, CES utility function is not suitable, because of its linear extension line which means that the proportion of income spent on each kind of goods is constant with the real income increasing. After comparing many models, a proper utility function derived from the CES utility function fits well. It is based on the constrained optimization of individuals, and consistent with the diminishing marginal utility, the Engel’s Law and the Theorem of Petty & Clarke.

Assume that the consumption ratio of GDP is constant in each year respectively, and the total consumption consists of both agricultural products and non-agricultural products purchases. The

4 In a CGE model, closure condition is the equation to be rejected in order to keep the consistency of the model.
aggregate consumption also includes the government purchases, which is made exogenous.

2.3 Other Factors of the Market

Assume that the duty of a good government is to maintain the market, and reduce the market distortion at a very low cost. All the impacts that the government has on the market are the administration expenditure and non-distortional taxes. It doesn’t matter whether this assumption contradicts the reality, as it is the government which often creates the distortion.

Capital was not considered here, thus capital market is omitted in the analysis. International trade has little effects on agriculture of China; it is also omitted for convenience.

In the long run, tendency is more important than fluctuation, thus labor unemployment rate is not so important in the long run. Assume that the number of workers is fixed, thus every unit of labor increase in one sector means one unit decrease in another. When marginal productivity of labor in the two sectors converges, equilibrium is achieved, and labor migration stops. This assumption of fixed employment is imposed because there is no credible statistical data of unemployment rate in China. It is impossible to take the labor supply function into consideration.

2.4 Model as a Whole

\[ Q_1 = A_1 K_1^{\alpha_1} L_1^{1-\alpha_1} \]
\[ Q_2 = A_2 K_2^{\alpha_2} L_2^{1-\alpha_2} \]
\[ (1-\alpha_1) P_1 A_1 (K_1/L_1)^{\alpha_1} = w_1' \]
\[ (1-\alpha_2) P_2 A_2 (K_2/L_2)^{\alpha_2} = w_2' \]
\[ \delta \frac{P_1 C_1^{\alpha_1-1}}{1-\delta} \frac{P_2 C_2^{\alpha_2-1}}{P_1} = \frac{P_1}{P_2} \]
\[ C_1 + G_1 = \lambda_1 Q_1 \]
\[ C_2 + G_2 = \lambda_2 Q_2 \]
\[ E = s(P_1 Q_1 + P_2 Q_2) \]
\[ P_1 C_1 + P_2 C_2 = E \]
\[ P_1 Q_1 + P_2 Q_2 = P(Q_1 + Q_2) \]
\[ L_1 + L_2 = L \]
\[ w_1' = w_2' \]
Research of Mathematical Economics No. 1 2008

This model simplified the circulation of the real economy. With the capital market, tax
distortion, and international trade excluded, this model concentrates on the labor reallocation and
output allocation. Output is determined by inputs in each sector, and wage rate by the marginal
productivity of labor. When some labor transfers, outputs changes, and so is the real consumption.
Price adjustment is the next one to change, and wage rate follows. In order to solve the
simultaneous equations, it is helpful to decompose the conducting process of price into steps.

Labor reallocation affects the output structure and then the consumption structure:

\[ C_1' = \lambda_1 A_1 K_1^{\alpha_1} L_1^{1-\alpha_1} - G_1 \]
\[ C_2' = \lambda_2 A_2 K_2^{\alpha_2} L_2^{1-\alpha_2} - G_2 \]

After consumption standard changes with the reallocation of labor, price adjusts as follows:

\[ P_1' = \frac{\delta \rho_1 C_1^{'p_1-1} P(Q_1 + Q_1')}{\delta \rho_1 C_1^{'p_1-1} Q_1 + (1-\delta) \rho_2 C_2^{'p_2-1} Q_2} \]
\[ P_2' = \frac{(1-\delta) \rho_2 C_2^{'p_2-1} P(Q_1' + Q_2')}{\delta \rho_1 C_1^{'p_1-1} Q_1 + (1-\delta) \rho_2 C_2^{'p_2-1} Q_2} \]

Wage rates also adjust to fit the new marginal productivity of labor level:

\[ w_1' = (1-\alpha_1) P_1 A_1 \left(K_1 / L_1\right)^{\alpha_1} \]
\[ w_2' = (1-\alpha_2) P_2 A_2 \left(K_2 / L_2\right)^{\alpha_2} \]

With labor in agricultural sectors switching to modern sectors, wage rates tend to converge.
When the wage rates converge, there will be equilibrium in the labor market.

Equations (1) (2) (3) are just for solving the simultaneous equations; real process is much
more complicated and might not fit the equilibrium conditions step by step. However, the
equilibrium only represents the potential tendency, which works in the long-run.

3. Data Processing and Parameter Estimation

3.1 Parameters Sorted

\[ G_1, G_2, \lambda_1, \lambda_2, P, \alpha_1, \alpha_2, A_1 K_1^\alpha, A_2 K_2^\alpha, \rho_1, \rho_2, \delta \]

are exogenous variables which can be derived from the national accounting indicators;
\[ \alpha_1, \alpha_2, A_1, K_1^\alpha, A_2 K_2^\alpha, \rho_1, \rho_2, \delta \]

are parameters in the production and utility functions which are to be estimated; \[ Q_1, Q_2, C_1, C_2, L_1, L_2, P_1, P_2, E, w_1, w_2, s \] are
Labor Market Distortion and Price Adjustment

endogenous variables which is to be calculated in the model. Here, \( \alpha_1, \alpha_2 \) only presents the elasticity of factors, but not the share of income of factors.

3.2 Indicators Selected

Considering the validation of macro indicators of China, time series from year 1991 to 2007 are selected. The following 12 indicators are sufficient and necessary to compute the model in this paper and 4 additional indicators are for further study:

<table>
<thead>
<tr>
<th>Categories</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1. Nominal GDP&lt;br&gt;2. Nominal Agricultural Product&lt;br&gt;3. Real GDP Index&lt;br&gt;4. Real Agricultural product Index</td>
</tr>
</tbody>
</table>

3.3 Evaluation of \( \gamma_1, \gamma_2, G_1, G_2, L, P \)

This step treats each year separately, and any intertemporal impacts are not considered.

Considering the consistency of statistic system, all of the raw data are taken from the official website of National Bureau of Statistics of China. Labor employed has been included in the raw data, while GDP Deflators can be derived from the nominal GDP and real GDP Index in each sector. Household and government consumption of agricultural products are given, and cannot be calculated from the current statistics directly. Some substitute measurements are available.

Firstly, calculate the re-input rate of agricultural products in some particular years according to the input-output tables of year 1987, 1992, 1995, 1997, 2000, and 2002. These products flowing to other sectors mostly turn into non-durable goods, so this part of agricultural products can be regarded as consumable. The re-input rates in the rest years are given by linear
Research of Mathematical Economics No. 1 2008

Interpolation.

Secondly, evaluates the government consumption of food. Though not very accurate, the regalement expenditure of the government in 2006 had gone up to ¥ 200 billion, which consists one third of the administration expenditure according to some estimation. Since this part of food consumption is much less compared with that of the households, holding the share for official regalement constant would not cause any significant errors. Then, government food consumption of each year can be estimated.

Thirdly, transfer the food data into those of agricultural products. Sum households and government food consumption, and compare this summation to the gross consumption of agricultural products. The exceeding part is value added to the raw products. Since the government share of food consumption is much less than that of the households, holding the ratio of added value constant might not cause significant errors either. Then, the households and government consumption of agricultural products can be inferred from food consumption and the additional value ratio. Gross data of the non-agricultural sectors equals to those of the nation subtract those of agricultural sectors. Then hold the consumption rates of each year constant.

3.4 Parameter Estimation for the Production Functions

Model: Cobb-Douglas Function

\[ Q_{i,t} = A_{i,t} \cdot e^{\beta (t-r_i)} K_{i,t}^\alpha L_{i,t}^{1-\alpha} \]

\[ \ln q_{i,t} = g_i (t - T_0) + \alpha_i \ln k_{i,t} + \varepsilon_i \]

Where

\[ q_{i,t} = \left( \frac{Q_{i,t}}{L_{i,t}} \right) / \left( \frac{Q_{0,t}}{L_{0,t}} \right), \quad k_{i,t} = \left( \frac{K_{i,t}}{L_{i,t}} \right) / \left( \frac{K_{0,t}}{L_{0,t}} \right) \]

Regression of agricultural sectors is not as easy as that of non-agricultural sectors, for the lack of agricultural capital formation data. This paper estimates the later one, and directly uses the results of Hongbin Zhao (2004) for the agricultural capital-output elasticity.

\[ \alpha_1 = 0.4556, \quad (t = 12.912, \ p = 0.000) \]
\[ \alpha_2 = 0.8208, \quad (t = 31.530, \ p = 0.000) \]

Then, calculate the non-labor factors in each year:

\[ A_{i,t} K_{i,t}^{\alpha_1} = Q_{i,t} L_{i,t}^{\alpha_2 - 1} e^{\beta (t-r_i)} \]
Labor Market Distortion and Price Adjustment

3.5 Parameter Estimation for the Utility Functions

First order condition of the optimization\(^5\)

\[
\frac{\delta}{1-\delta} \cdot \frac{P_1}{P_1^{\rho_1}} \cdot \frac{C_{\rho_1}}{C_{\rho_1}^{\rho_1}} = \frac{P_2}{P_2^{\rho_2}}
\]

\[
\frac{\delta}{1-\delta} \cdot \frac{P_1}{P_2^{\rho_2}} \cdot \frac{P_2^{\rho_2}}{P_1^{\rho_1}} \cdot \frac{\eta^{\rho_1-1}}{(1-\eta)^{\rho_2-1}} \cdot E^{\rho_1-\rho_2} = 1
\]

The main property of Chinese households’ utility function in the long-run is the diminishing Engel Coefficients, which is the result of different elasticity of agricultural products and non-agricultural products. This model adds one more degree of freedom to the original CES function, so another degree of freedom should be taken down in order to estimate the parameters.

Firstly, hold \(P_2^{\rho_2} / P_1^{\rho_1}\) constant, and acquire the relationship of \(\rho_1, \rho_2\). Secondly, minimize \(P_2^{\rho_2} / P_1^{\rho_1}\) subject to the relationship of \(\rho_1, \rho_2\). Then, \(\rho_1, \rho_2\) are fixed.

This measurement is based on three properties. The first one is that monotonic transformation of utility function does not change the optimal solution. The second one is that the relative prices of China vary little in the latest 17 years. The third one is that the price adjustment is much more immediate than that of supply.

Take logarithm of equation (7):

\[
\ln E = \beta_1 \ln \eta + \beta_2 \ln (1-\eta) + \beta_3 + \varepsilon
\]

Where

\[
\beta_1 = \frac{\rho_1 - 1}{\rho_2 - \rho_1}, \beta_2 = -\frac{\rho_2 - 1}{\rho_2 - \rho_1}, \beta_3 = \frac{1}{\rho_2 - \rho_1} \ln \left( \frac{\delta}{1-\delta} \cdot \frac{P_1}{P_2^{\rho_2}} \cdot \frac{P_2^{\rho_2}}{P_1^{\rho_1}} \right)
\]

\[
\delta = \left[ \left( \frac{e^{\beta_1 (\rho_2 - \rho_1)} \cdot P_1^{\rho_1}}{P_2^{\rho_2}} \cdot \frac{P_2^{\rho_2}}{P_1^{\rho_1}} \right)^{-1} + 1 \right]^{-1}
\]

Constraint Regression:

\[
\ln E = -2.5948 \ln \eta + 1.5948 \ln (1-\eta) + 7.9519
\]

\[
(0.000) \quad (0.000) \quad (0.000)
\]

Constraint Optimization:

\[\text{-----------------------------------------------}\]

\(^5\) The main deduction is in the appendix.
\[
\begin{aligned}
Min & \left\{ \frac{\text{Std} \left( \rho_2 \ln P_2 - \rho_1 \ln P_1 \right)}{\rho_1 \rho_2} \right\} \\
\text{s.t.} & \quad (1 + \beta_1) \rho_1 - \beta_1 \rho_2 = 1 \\
& \quad \rho_1 = 0.5576, \quad \rho_2 = 0.7281, \quad \bar{\delta} = 0.7404
\end{aligned}
\]

3.6 The robustness of this model

In order to avoid the instability of nonlinear system, this paper cuts the simultaneous equations where it is nonlinear, and uses iteration algorithm which considers both mathematical convenience and explicit economic implications. The potential instability producers are:

\[\alpha_1, \alpha_2, \beta_1, \beta_2\]

Actually, it does not have significant effects on the equilibrium to introduce disturbance of normal distribution with deviation equaling to 10% of the values. What’s more, even if the value of the outcome is different, the long run tendency with specific characteristics is still significant.

However, this model, which lacks dynamic foundations, is not able to export smooth series. Consequently, some turning point of the process seems to be a little abrupt such as the year 2004.

4. Solving the Model

4.1 Algorithm

This algorithm simulates the migration of labor, and the equilibrium condition is \(w1 = w2\).
4.2 Data Imported and Data Exported

Import all of the exogenous variables and parameters estimated, and export endogenous variables including the real output level, employed labor, price level, consumption and wage rates of the two sectors in the equilibrium. Then, compare the real economy to the equilibrium level, and calculate the optimal adjusting rates of the labor market and price level.

Actually, even in most developed countries, \( w_1 \neq w_2 \), because of the immanent property of agriculture and government subsidies. Therefore, it is not necessary to have marginal productivity of labor in the two sectors equal, say, \( w_1 = k \cdot w_2 \) is okay. This paper calculates the equilibrium with \( k = 100\%, 90\%, 80\% \) respectively, and illustrates the case when \( k = 80\% \):
Graph 2 The Optimal Adjusting Rate of Labor when \( w_1 = 70\% w_2 \)

Graph 3 The Optimal Adjusting Rate of Price when \( w_1 = 80\% w_2 \)
5. Additional Evidence

In order to have a deeper look at the economic structure upgrading in China, this chapter analyzes the growth of the labor productivity, and provide some other evidence which presents similar results as the CGE model.

According to the production function in this paper, both the labor and non-labor factors have effects on the labor productivity. Output grows when input grows, but labor productivity decreases as the labor employed increases. The growth rate related to labor equals to that of labor productivity subtracts that of non-labor factors. In agricultural sectors, the labor employed is decreasing, thus the effect is positive; while in non-agricultural sectors, it is inversed. Therefore, if there was no labor transferring, the disparity in labor productivity would be even larger.

There is another interesting phenomenon. As Labor productivity Growth in non-agricultural sectors is faster than those of agricultural sectors. the aggregate growth rate should be somewhat between those of the two sectors intuitively. However, it is not the truth in China. In some years, the aggregate growth rate is even high than each of the two sectors. It is not because of the errors in statistical data, but the effect of economic structure upgrading. The change of the relative sizes of the two sectors also contributes to the aggregate growth.

Graph 4 Contribution of labor employed on labor productivity growth
6. Empirical Analysis

The purpose of this paper is to quantify the distortion of labor market and price in China within a general equilibrium framework, and judge which developing stage China is in now according to the long-run tendency of market distortion.

From Graph 2 to Graph 4, the labor productivity of the two sectors (not presented here), and main economic policies of the government, it is intuitive to divide the year 1991–2007 into three distinct parts.

The first one is 1991–1997 when market reform began to influence the economy all over China. In this period, the market was not stable, and the structural inflation and government directed inflation waved the market violently. During those 7 years, the rapid growth of industry attracted a great amount of peasants, and the productivity of labor in these two sectors grew at the almost same pace. There was a wave crest of urban-rural disparity in 1994, and the crest fell immediately in the next two years. It was determined by the government directed inflation. In those years, the centre government cut the fiscal income to stimulate the economy, while the fiscal expenditure did not decrease relatively to match the deceasing income. In order to deal with the substantive deficit, the centre government had to emit a large amount of money, which resulted in the reallocation of social wealth to a great extent. After the year 1995, the violent inflation was gradually under control, and the urban-rural disparity fell responding to it. As a whole, the relative living condition of rural household didn’t deteriorate despite some fluctuations.
Labor Market Distortion and Price Adjustment

The second one is 1998–2003 when the Asian Financial Crisis took place. In this period, the price level kept on falling and the labor demand was low, thus few agricultural labor transferred to the industry. As a result, the labor productivity of agriculture almost stayed at the original level at the end of this period. What’s more, during these 6 years, the food supplies are sufficient, and the price of agricultural products even decreased in 2 of the years. Since the regulation of labor market had been released during these years, it is more proper to regard this period as the eve before industry conversely nurturing agriculture, rather than the period when agriculture sacrifices to support industry.

The last period is 2004–now, when the government attempts to conversely nurture agriculture. In this period, agriculture taxes are abandoned and the “New Village Construction” takes place around China. Labor market barriers are lowered, and the government begins to take measure to weaken the influence of “Hukou” system. The price of agriculture products rose by more than 20% two times respectively in 2004 and 2007. Considering the condition as a whole, the need to adjust price has reduced remarkably and the relative labor surplus in rural areas does not increase during the latest 5 years. This nurture is a good beginning, but not sufficient. The surplus labor in rural areas is under control, but far from decreasing.

7. Conclusions and Extensions

The work done in this paper is not quite complex, and even lack of accuracy. The World Bank has done a lot of work on the policy, but few researchers have done the work on the long run economic system. Many non-market factors have impacts on the equilibrium, especially in China. However, in order to make a little progress in this field, it is necessary to make some compromise by introducing some rigorous assumptions, such as the small government assumption.

Regardless of all those shortcomings, this paper summarizes as follows:

1. Labor Market Distortion: Involution is one of the most serious problems in Chinese agriculture. In 2007, the nominal surplus of labor in agriculture is at least 28%. When this amount of labor has successfully transferred to non-agricultural sectors, the price of agricultural products should increase by 6% correspondingly. After 2004, the increasing labor surplus is under control, while its proportion is still too high.

2. Labor Transferring: Moving off Agriculture is the long run tendency of Chinese process of
industrialization and modernization, despite some reversed years. Labor migration has prevented the urban-rural disparity to some extent, though not enough.

3. Structural Upgrading: Inner growth in each sector is the main cause of growth while structure upgrading explains more than one quarter of the growth rate. This contribution ratio fluctuates in the short run but stays on a relative stable level in the long run.

4. Three Stages: 1991–1997 is the first one when agriculture benefits equally from the reform. 1998–2003 is the period when agriculture is significant weak compared to other sectors. 2004–now is the last one when industry conversely nurtures agriculture.

In the sequential research, the most difficult task is to overcome Lucas Critique. Lucas (1976) said “It is naive to try to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data, especially highly aggregated historical data.” In order to avoid this problem, the CGE model should at least be developed into dynamic ones, taking rational expectation into consideration.

The next big shortage of this paper is the small government assumption which is quite different from "the Chinese Model". In this paper, this assumption is imposed for convenience, but in the real economy, the government often plays a important role, especially in the economic transformation.

The assumption of homogenous households and close economy can be released by dividing the households into different groups. The assumption of homogenous labor force contradicts China’s real economy seriously; however, there are no sufficient valid data to distinguish the labor force. Additionally, effective empirical work on economics should provide powerful and feasible instruction for Policy making, which is also not done in this paper.

**Appendix**

**The Utility Function**

Assume that consumers are homogenous and the average utility function is:

\[ U = \delta C_1^{\phi} + (1 - \delta) C_2^{\rho}; \]

There is a little but vital difference between CES function and this function, because the substitute elasticity is not constant in this function:
Labor Market Distortion and Price Adjustment

\[ \sigma = \frac{d \ln \left( \frac{C_1}{C_2} \right)}{d \ln \left( \frac{U_1}{U_2} \right)} = \frac{d \ln \left( \frac{C_1}{C_2} \right)}{d \ln \left[ \left( \rho_1 (1-\delta) C_2^{\rho_2} \right) \left( \rho_1 \delta C_1^{\rho_2} \right) \right]} \]

\[ = \frac{1}{(1-\rho_1)} \cdot \frac{d \ln C_1}{d \ln C_2} \frac{d \ln C_2}{d \ln C_1} \]

Constraint Optimization to Equation (11):

\[ \max_{C_1, C_2} U = \delta C_1^{\rho_1} + (1-\delta) C_2^{\rho_2} \]
\[ \text{s.t.} \ P_1 C_1 + P_2 C_2 = E \]

The first order condition is:

\[ \frac{\delta}{1-\delta} \cdot \frac{\rho_1 C_1^{\rho_2-1}}{\rho_2 C_2^{\rho_2-1}} = \frac{P_1}{P_2} \]

Let \( \eta = P_1 C_1 / E \), and then \( C_1 = \eta E / P_1 \), \( C_2 = (1-\eta) E / P_2 \), and Substitute them into equation (12):

\[ \frac{\delta}{1-\delta} \cdot \frac{\rho_1 P_1^{\rho_1}}{\rho_2 P_2^{\rho_2}} \cdot \frac{\eta^{\rho_1-1}}{(1-\eta)^{\rho_2-1}} \cdot E^{\rho_1-\rho_2} = 1 \]

Take logarithm:

\[ \ln \left( \frac{\delta}{1-\delta} \cdot \frac{\rho_1 P_1^{\rho_1}}{\rho_2 P_2^{\rho_2}} \right) + (1-\rho_2) \ln C_2 - (1-\rho_1) \ln C_1 = \ln \left( \frac{P_1}{P_2} \right) \]

Take derivative:

\[ (1-\rho_2) d\ln C_2 - (1-\rho_1) d\ln C_1 = d\ln \left( \frac{P_1}{P_2} \right) \]

Hold \( P_1 / P_2 \) constant and let \( 0 < \rho_1 < \rho_2 < 1 \):

\[ \frac{d\ln C_2}{d\ln C_1} = \frac{1-\rho_2}{1-\rho_1} > 1 \]

Therefore, with the relative price constant, the growth rate of non-agricultural products consumption is larger than that of agricultural products, which means that the percentage households spend on agricultural products decreases as the real income increases.

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>L1</th>
<th>L2</th>
<th>P1</th>
<th>P2</th>
<th>C1</th>
<th>C2</th>
<th>w1</th>
<th>w2</th>
<th>E</th>
<th>s</th>
<th>Engel*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>4746</td>
<td>17206</td>
<td>31463</td>
<td>34028</td>
<td>1.079</td>
<td>0.978</td>
<td>3903</td>
<td>6840</td>
<td>886</td>
<td>886</td>
<td>10903</td>
<td>49.7%</td>
<td>38.6%</td>
</tr>
<tr>
<td>1992</td>
<td>4786</td>
<td>20294</td>
<td>29061</td>
<td>57091</td>
<td>1.162</td>
<td>1.063</td>
<td>4028</td>
<td>8042</td>
<td>1042</td>
<td>1042</td>
<td>13231</td>
<td>48.8%</td>
<td>35.4%</td>
</tr>
</tbody>
</table>

90
Research of Mathematical Economics No. 1 2008

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4898</td>
<td>5132</td>
<td>5499</td>
<td>5841</td>
<td>5895</td>
<td>6002</td>
<td>6003</td>
<td>5934</td>
<td>5955</td>
<td>5991</td>
<td>5983</td>
<td>6531</td>
<td>6801</td>
<td>7107</td>
<td>7428</td>
</tr>
<tr>
<td></td>
<td>23647</td>
<td>27129</td>
<td>30239</td>
<td>33451</td>
<td>37091</td>
<td>40317</td>
<td>43958</td>
<td>48389</td>
<td>52963</td>
<td>58448</td>
<td>65176</td>
<td>71626</td>
<td>79528</td>
<td>89326</td>
<td>100568</td>
</tr>
<tr>
<td></td>
<td>27139</td>
<td>26735</td>
<td>26925</td>
<td>26903</td>
<td>25704</td>
<td>25185</td>
<td>24340</td>
<td>22988</td>
<td>22274</td>
<td>21582</td>
<td>20403</td>
<td>20661</td>
<td>19519</td>
<td>18550</td>
<td>18177</td>
</tr>
<tr>
<td></td>
<td>39669</td>
<td>40720</td>
<td>41140</td>
<td>42047</td>
<td>44115</td>
<td>45452</td>
<td>47054</td>
<td>49097</td>
<td>50751</td>
<td>52158</td>
<td>54029</td>
<td>54539</td>
<td>56306</td>
<td>57850</td>
<td>58813</td>
</tr>
<tr>
<td></td>
<td>1.335</td>
<td>1.679</td>
<td>1.969</td>
<td>2.129</td>
<td>2.167</td>
<td>2.178</td>
<td>2.188</td>
<td>2.254</td>
<td>2.350</td>
<td>2.441</td>
<td>2.553</td>
<td>2.754</td>
<td>2.802</td>
<td>2.891</td>
<td>3.152</td>
</tr>
<tr>
<td></td>
<td>1.228</td>
<td>1.470</td>
<td>1.662</td>
<td>1.765</td>
<td>1.796</td>
<td>1.778</td>
<td>1.755</td>
<td>1.794</td>
<td>1.829</td>
<td>1.837</td>
<td>1.866</td>
<td>2.014</td>
<td>2.100</td>
<td>2.180</td>
<td>2.289</td>
</tr>
<tr>
<td></td>
<td>4070</td>
<td>4198</td>
<td>4454</td>
<td>4764</td>
<td>4830</td>
<td>4916</td>
<td>4913</td>
<td>4838</td>
<td>4788</td>
<td>4722</td>
<td>4672</td>
<td>4969</td>
<td>5256</td>
<td>5475</td>
<td>5746</td>
</tr>
<tr>
<td></td>
<td>9148</td>
<td>10167</td>
<td>11811</td>
<td>13476</td>
<td>14755</td>
<td>16087</td>
<td>17928</td>
<td>19855</td>
<td>21195</td>
<td>22880</td>
<td>24454</td>
<td>24969</td>
<td>27206</td>
<td>30046</td>
<td>32689</td>
</tr>
<tr>
<td></td>
<td>1311</td>
<td>1755</td>
<td>2189</td>
<td>2516</td>
<td>2706</td>
<td>2826</td>
<td>2938</td>
<td>3168</td>
<td>3421</td>
<td>3689</td>
<td>4077</td>
<td>4740</td>
<td>5314</td>
<td>6031</td>
<td>7013</td>
</tr>
<tr>
<td></td>
<td>16663</td>
<td>21990</td>
<td>28396</td>
<td>33925</td>
<td>36969</td>
<td>39314</td>
<td>42211</td>
<td>46520</td>
<td>50027</td>
<td>53552</td>
<td>58047</td>
<td>64317</td>
<td>71850</td>
<td>81316</td>
<td>92924</td>
</tr>
<tr>
<td></td>
<td>46.8%</td>
<td>45.4%</td>
<td>46.5%</td>
<td>47.5%</td>
<td>46.6%</td>
<td>46.4%</td>
<td>46.8%</td>
<td>46.4%</td>
<td>45.1%</td>
<td>43.9%</td>
<td>42.0%</td>
<td>39.6%</td>
<td>38.6%</td>
<td>37.8%</td>
<td>36.6%</td>
</tr>
<tr>
<td></td>
<td>32.6%</td>
<td>32.0%</td>
<td>30.9%</td>
<td>29.9%</td>
<td>28.3%</td>
<td>27.2%</td>
<td>25.5%</td>
<td>23.4%</td>
<td>22.5%</td>
<td>21.5%</td>
<td>20.6%</td>
<td>21.8%</td>
<td>20.5%</td>
<td>19.5%</td>
<td>19.5%</td>
</tr>
</tbody>
</table>

* The original definition of Engel’s Coefficient has been modified in order to keep consistent with other data in this paper.

Table 2: Distortion rates of labor market and price adjusting rates in each year at 4 levels

<table>
<thead>
<tr>
<th>Year</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
<th>L1</th>
<th>L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>-19.5</td>
<td>28.9</td>
<td>-14.0</td>
<td>20.7</td>
<td>-7.9</td>
<td>11.7</td>
<td>-1.8</td>
<td>-3.5</td>
<td>-7.9</td>
<td>11.7</td>
<td>-1.8</td>
<td>-3.5</td>
<td>-7.9</td>
<td>11.7</td>
<td>-1.8</td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>-24.9</td>
<td>35.1</td>
<td>-19.2</td>
<td>27.1</td>
<td>-12.9</td>
<td>18.2</td>
<td>-7.0</td>
<td>1.8</td>
<td>-22.6</td>
<td>26.1</td>
<td>-18.5</td>
<td>18.6</td>
<td>-5.5</td>
<td>0.7</td>
<td>-2.8</td>
<td></td>
</tr>
<tr>
<td>1993</td>
<td>-28.0</td>
<td>36.2</td>
<td>-22.1</td>
<td>28.7</td>
<td>-15.6</td>
<td>20.2</td>
<td>8.3</td>
<td>1.9</td>
<td>-27.0</td>
<td>32.1</td>
<td>-14.2</td>
<td>16.9</td>
<td>3.0</td>
<td>-0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>-27.0</td>
<td>32.1</td>
<td>-21.0</td>
<td>24.9</td>
<td>-10.8</td>
<td>11.8</td>
<td>0.0</td>
<td>0.2</td>
<td>-25.2</td>
<td>33.2</td>
<td>-16.3</td>
<td>16.6</td>
<td>0.5</td>
<td>0.6</td>
<td>-28.4</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>-24.2</td>
<td>26.4</td>
<td>-17.9</td>
<td>19.6</td>
<td>-10.8</td>
<td>11.8</td>
<td>0.0</td>
<td>0.2</td>
<td>-22.7</td>
<td>23.2</td>
<td>-16.3</td>
<td>16.6</td>
<td>0.5</td>
<td>0.2</td>
<td>-22.7</td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>-26.2</td>
<td>26.1</td>
<td>-19.8</td>
<td>19.7</td>
<td>-12.4</td>
<td>12.4</td>
<td>0.4</td>
<td>0.1</td>
<td>-28.4</td>
<td>28.2</td>
<td>-22.0</td>
<td>21.8</td>
<td>4.3</td>
<td>-0.4</td>
<td>-8.9</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>-32.0</td>
<td>32.1</td>
<td>-25.7</td>
<td>25.8</td>
<td>-18.5</td>
<td>18.6</td>
<td>5.5</td>
<td>0.7</td>
<td>-28.4</td>
<td>28.2</td>
<td>-22.0</td>
<td>21.8</td>
<td>4.3</td>
<td>-0.4</td>
<td>-14.7</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>-36.2</td>
<td>36.2</td>
<td>-30.1</td>
<td>30.1</td>
<td>-23.0</td>
<td>23.0</td>
<td>9.8</td>
<td>-1.3</td>
<td>-32.0</td>
<td>32.1</td>
<td>-25.7</td>
<td>25.8</td>
<td>8.1</td>
<td>-1.0</td>
<td>-18.5</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>-39.0</td>
<td>39.0</td>
<td>-33.0</td>
<td>33.0</td>
<td>-26.1</td>
<td>26.1</td>
<td>11.3</td>
<td>-1.4</td>
<td>-41.5</td>
<td>41.5</td>
<td>-35.6</td>
<td>35.6</td>
<td>16.5</td>
<td>-1.7</td>
<td>-28.8</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>-41.5</td>
<td>41.5</td>
<td>-32.3</td>
<td>32.3</td>
<td>-26.1</td>
<td>26.1</td>
<td>11.3</td>
<td>-1.4</td>
<td>-44.2</td>
<td>42.6</td>
<td>-38.4</td>
<td>37.1</td>
<td>18.5</td>
<td>-1.7</td>
<td>-31.6</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>-44.2</td>
<td>42.6</td>
<td>-35.4</td>
<td>34.5</td>
<td>-28.3</td>
<td>25.0</td>
<td>7.2</td>
<td>-0.4</td>
<td>-41.4</td>
<td>36.6</td>
<td>-35.4</td>
<td>34.5</td>
<td>10.2</td>
<td>-0.6</td>
<td>-29.2</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>-41.4</td>
<td>36.6</td>
<td>-31.3</td>
<td>31.3</td>
<td>-28.3</td>
<td>25.0</td>
<td>7.2</td>
<td>-0.4</td>
<td>-42.5</td>
<td>34.5</td>
<td>-36.4</td>
<td>29.6</td>
<td>12.4</td>
<td>-0.9</td>
<td>-29.2</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>-42.5</td>
<td>34.5</td>
<td>-31.3</td>
<td>31.3</td>
<td>-28.3</td>
<td>25.0</td>
<td>7.2</td>
<td>-0.4</td>
<td>-42.5</td>
<td>34.5</td>
<td>-36.4</td>
<td>29.6</td>
<td>12.4</td>
<td>-0.9</td>
<td>-29.2</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>-42.5</td>
<td>34.5</td>
<td>-31.3</td>
<td>31.3</td>
<td>-28.3</td>
<td>25.0</td>
<td>7.2</td>
<td>-0.4</td>
<td>-42.5</td>
<td>34.5</td>
<td>-36.4</td>
<td>29.6</td>
<td>12.4</td>
<td>-0.9</td>
<td>-29.2</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>-42.5</td>
<td>34.5</td>
<td>-31.3</td>
<td>31.3</td>
<td>-28.3</td>
<td>25.0</td>
<td>7.2</td>
<td>-0.4</td>
<td>-42.5</td>
<td>34.5</td>
<td>-36.4</td>
<td>29.6</td>
<td>12.4</td>
<td>-0.9</td>
<td>-29.2</td>
<td></td>
</tr>
</tbody>
</table>
Labor Market Distortion and Price Adjustment

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>-43.0%</td>
<td>-42.2%</td>
<td></td>
</tr>
<tr>
<td>-32.0%</td>
<td>-29.1%</td>
<td></td>
</tr>
<tr>
<td>16.5%</td>
<td>12.5%</td>
<td></td>
</tr>
<tr>
<td>-1.1%</td>
<td>-0.5%</td>
<td></td>
</tr>
<tr>
<td>-36.8%</td>
<td>-35.8%</td>
<td></td>
</tr>
<tr>
<td>27.4%</td>
<td>24.7%</td>
<td></td>
</tr>
<tr>
<td>13.3%</td>
<td>9.4%</td>
<td></td>
</tr>
<tr>
<td>-1.0%</td>
<td>-0.4%</td>
<td></td>
</tr>
<tr>
<td>-29.5%</td>
<td>-28.3%</td>
<td></td>
</tr>
<tr>
<td>21.9%</td>
<td>19.5%</td>
<td></td>
</tr>
<tr>
<td>10.0%</td>
<td>6.1%</td>
<td></td>
</tr>
<tr>
<td>-0.8%</td>
<td>-0.2%</td>
<td></td>
</tr>
</tbody>
</table>

References

English Literature:


Ian Sue Wing, “Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis”.

Shantayanan Devarajan, Delfin S. Go, Jeffrey D. Lewis, Sherman Robinson, Pekka Sinko, “Simple General Equilibrium Modeling”.


Chinese Literature:


(Responsible Editor: Zhang Lei) (Proofreader: Lu Hong)
Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium

Zhang Lei
(School of Economics, Renmin University of China)

Abstract: In a non-stochastic economy with identical infinitely-lived agent, Chamley (1986) and Judd (1985) have shown that the optimal policy is eventually to set the capital income tax to zero. This paper shows that for Chamley’s model with investment tax credit, the optimal tax rate on capital income is equal to rate of investment tax credit; therefore the tax can be positive and non-distortionary, even in the long run. Finally, I conclude that government can collect substantial positive revenue under two conditions: after-tax gross rate of return on bonds is higher than the capital stock growth rate, and bounds must be imposed on the initial tax rates of both capital income and investment credit.

Key words: capital income tax; investment tax credit; general equilibrium

1. Introduction

In public finance, a classical question is how to set the rate of those taxations to collect revenue in a least distortionary way, which is also called the taxation of the second best. One particularly interesting aspect of this question is the redistributive potential of capital income taxation: how much disincentive effects of capital income tax on capital accumulation and the resulting loss in wages reduce total social welfare. Previous studies of the optimal capital income taxation indicate different results. Some studies, using neoclassical growth model in a dynamic general equilibrium by Feldstein (1974), Grieson (1975), Boulding (1979), Bernheim (1981), Homma (1981), and among others, indicate that the optimal capital income tax rate is not zero. The major shortcoming of these studies is their limited scope for intertemporal substitution. When taking the intertemporal substitution effect into consideration, Chamley (1986) and Judd (1985),

1 I am indebted to Leslie Reinshorn for many helpful insights and suggestions that have substantially improved this paper. And I also thank for helpful comments of the students at Renmin University-Doshisha University Joint Seminar and seminar at Waseda University. The views expressed herein and all errors are my own.
using Ramsey framework with a representative infinitely-lived household, find a celebrated result that the optimal tax rate on capital income is zero in the long run. Chari, Christiano, and Kehoe (1994) perform numerical simulations and conclude that there is a quantitative presumption that the *ex ante capital tax rate* is approximately zero.

Lucas (1990) estimates for the U.S. capital income tax rate was about 36 percent, which was far from zero. Moreover, he, using a representative agent model, argues that for the U.S. economy there is a significant welfare gain from eliminating the capital income tax. Recent studies mainly focus on why the capital income tax is so high in reality, which is contrary to Chamley-Judd results. If there is a stationary Ramsey allocation, Zhu (1992) shows that there are two possible outcomes. For some special utility functions, the Ramsey plan prescribes a zero ex ante capital tax rate that can be implemented by setting a zero tax on capital income. But except for special classes of preferences, Zhu concludes that the ex ante capital tax rate should vary around zero, in the sense that there is a positive measure of states with both positive and negative tax rates. Aiyagari (1995) shows that with incomplete markets, borrowing constraints, and constant discounting, the optimal tax rate on capital income was positive. Lansing (1999) suggests that indeterminacy may arise when government had no ability to borrow; hence capital income taxes in the long run are indeterminate; in other words, the optimal capital income tax rate is non-zero in the long run. Abel (2007) proves that if purchasers of capital are permitted to deduct capital expenditure from taxable capital income, then a constant tax rate on capital income is optimal. Chien and Lee (2007) provide a generalized framework that when there is any endogenous discrepancy between government’s discount factor and that of the private sector, tax capital income is optimal.

Tax law generally allows owners of capital to reduce their calculated taxable income by some amount to reflect the cost of acquiring capital. This deduction in taxable income is usually implemented through depreciation of the capital asset (see Wikipedia, tax deduction). Traditional optimal capital income tax literatures often omit such deprecations allowance or investment tax credit. Extending the results of Abel (2007), this paper provides a counter-example to Chamley-Judd claim. In particular, I use the Chamley’s model to show that the optimal tax on optimal capital income is equal to rate of investment tax credit, thus can be non-zero, when government implement investment tax credit at the rate of $\tau'$.

---

2 The current market value of next period’s tax payments across states of nature, dividing by the current market value of capital income gives a measure that we call the *ex ante capital tax rate*. 

95
Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium

Before laying out the details of the counterexample, it is worthwhile to review the previous studies on investment tax credit. Lucas (1990, p300) mentions that “a tax on capital income combined with an investment tax credit can imitate a capital levy perfectly” and thus is non-distortionary. However, he doesn’t prove his statement. Besides, Fullerton, Henderson (1989) also shows that investment tax credit removes many inter-asset distortions while it may exacerbate small inter-sector distortions. Additionally, Bandy (1978) comprehensively discusses six investment incentives, including investment allowances, initial allowance, gross investment tax credit, net investment tax credit, accelerated depreciation, interest subsidy. Within strict confines of the neoclassical theory of investment, he shows that investment allowances and gross investment tax credits above regular depreciation are efficient incentives. But, initial allowances, net and gross investments tax credits, which are set against depreciation, and interest subsidies all distort investment decisions. Accelerated depreciation schemes are generally distortionary as well, with the nature of the distortion depending upon how the rate of depreciation tax is defined.

The economic intuition for the positive capital income taxation is straightforward. Because investment tax credits over and above regular depreciation decreases its marginal rate of substitution, there is an incentive for agents to accumulate capital instead of consumption and bonds purchase. In addition, the possibility of being borrowing-constrained in some future periods leads agents to accumulate capital. These two features lead to increases in their saving and hence capital accumulation and thereby lower the return on capital below the rate of time preference.

Another way to understand the intuition behind such tax policy is to note that government also has incentives to levy such tax policy if this tax policy can collect substantial revenue and spread out its redistribution policy, when it satisfies the condition that capital income is higher than investment at any arbitrary time. Intuitively, such condition can be reached and explained by the empirical evidence. In a closed economy, gross domestic income can be divided into two parts: capital income and labor income; otherwise, it can also be divided into three parts: consume, investment and government purchase. Econometrica regression indicates that capital income share is about 1/3 with Cobb-Douglas production function, and investment share of GDP in U.S is about 20%, lower than capital income.

Additionally, when regarding confiscatory taxation, this is something we have to look out for in any dynamic model of optimal taxation. If there is a stock variable that is subject to a tax (and subsidy), the optimal policy is to set a very high tax rate (and very negative subsidy rate) for a very
short period of time. This raises issues of time consistency, so we are supposed to impose an upper bound on the tax rate (and lower bound on the subsidy rate).

The rest of this paper is organized as follows. Section 2 presents basic Ramsey- Cass-Koopmans model and finds out the optimal rates of capital income tax and investment tax credit by Lagrangian method. Section 3 calculates the amount of tax revenue, at any arbitrary time and steady state as well as alone the balanced growth path. Section 4 discusses the bounds on initial tax rate of capital income and investment credit. Finally, section 5 presents my concluding remarks.

2. Model

This section presents the standard Ramsey-Cass-Koopmans model consists three types of agents: households, firms, and government. And then I calculate the optimal tax policy.

2.1 Firm

Consider a closed economy which each competitive firm is a price-taker to rent service of capital and labor in competitive markets. And firms also sell goods on perfect competitive markets. Labor is supplied by a continuous of identical infinitely-lived households. Each household works $L(t)$ hours at time $t$. The production function is

$$Y(t) = F(K(t), A(t)L(t))$$

Where $Y(t)$ is output, $K(t)$ is the capital stock, and $A(t)$ is an index of labor- augmenting technical progress that evolves over time. So $A(t)L(t)$ is the amount of effective hours of labor input. A standard concave production function $F(K, AL)$ exhibits constant returns to scale. By Euler’s theorem, linear homogeneity of production function implies $F(K, AL) = KF_1 + ALF_2$, and $F_1>0, F_2<0, F_{12}>0, F_{22}<0$.

The identical firm maximizes their profits:

$$MaxF(K(t), A(t)L(t)) - w(t)L(t) - r(t)K(t)$$

The first order condition with respect to labor and capital as follows:

$$w(t) = A(t)F_2(K(t), A(t)L(t))$$

$$r(t) = F_1(K(t), A(t)L(t))$$

In a word, inputs should be employed until marginal product of last unit is equal to its rental price. With constant returns to scale, I get the standard result that pure profits are zero and the size
Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium

of an individual firm is indeterminate.

Output can be consumed by households, used by government, or used for investment. The gross domestic product by expenditure approach is

$$C + G + I = F(K, AL)$$  \hspace{1cm} (1)

Assuming the capital depreciation rate, $\delta$, is constant over time, so the accumulation of capital over time is given by

$$K(t) = I(t) - \delta K(t)$$

Where $K(t)$ is gross investment at time $t$, and $0 < \delta < 1$.

2.2 Government

Government’s goal is to maximize households’ welfare, subjected to raising set revenues through tax system. When designing an optimal policy, government takes account equilibrium reactions by consumers and firms into tax system. Government finances its stream of purchases $G(t)$ by levying flat-rate, time-varying taxes on earnings from capital at rate $\tau^c(t)$ and from labor at rate $\tau^l(t)$ at time $t$. And government also gives the investment tax credit at rate $\tau^i(t)$. Government can also trade one-period bonds, sequential trading of which suffices to accomplish any intertemporal trade in a world without uncertainty. Let $B(t)$ be government indebtedness to the private sector, maturing at time $t$ with the return rate of $R(t)$. Government’s budget constraint is

$$G(t) + R(t)B(t) = T(t) + \dot{B}(t)$$  \hspace{1cm} (2)

The $T(t)$ is total tax revenue at time $t$, which can be separated into three parts: wage income tax, capital income tax and investment tax credit expenditure.

$$T(t) = \tau^c(t)K(t)r(t) - \tau^l(t)I(t) + \tau^i(t)w(t)L(t)$$

Where $w(t)$ is the wage rate for labor at time $t$.

2.3 Household

An infinitely lived representative household likes consumption, leisure streams $\{C(t), M(t)\}$ that give higher values of

$$\int_0^{\infty} e^{-\beta t}U(C(t), M(t))dt$$  \hspace{1cm} (3)

If the government expenditure was included into the utility function, the results regard with tax policy does not change, so here for simplicity I excluded government expenditure form utility function.
Where $U$ is increasing, strictly concave, and two times continuously differentiable in $C(t)$ and $M(t)$. $\beta$ is the time preference rate, which means preference of households for current consumption is relative to future consumption. Assume the household is endowed with one unit of time that can be used for leisure $M(t)$ and labor $L(t)$,

$$M(t) + L(t) = 1$$

Besides, assume the growth of population is zero. The single good is produced with labor $L(t)$ and capital $K(t)$. So, the representative household chooses the sequences of consumption, $C(t)$, hours of work, $L(t)$, capital, $K(t)$, and bonds, $B(t)$, to maximize the life-time utility subject to the following sequence of budget constraints:

$$\dot{B}(t) + J(t) + C(t) = (1 - \tau^L(t))w(t)L(t) + (1 - \tau^K(t))r(t)K(t) + \tau^L(t)J(t) + R(t)B(t)$$

(4)

And capital accumulation constraints:

$$\dot{K}(t) = J(t) - \delta K(t)$$

(5)

For this dynamic optimization problem, we can write the Hamiltonian function:

$$H = U(C,1 - L) + \lambda((1 - \tau^L)w + (1 - \tau^K)r + (\tau^L - 1)J + RB - C) + \mu(I - \delta K)$$

The first order conditions with respect to consumption, labor and investment are:

$$\frac{\partial H}{\partial C} = U_1(C,1 - L) - \lambda = 0$$

$$\frac{\partial H}{\partial L} = -U_2(C,1 - L) + \lambda(1 - \tau^L)w = 0$$

$$\frac{\partial H}{\partial I} = \lambda(\tau^L - 1) + \mu = 0$$

Euler equations of this problem are:

$$\dot{\lambda} - \beta \lambda = -\frac{\partial H}{\partial B} = -\lambda R$$

(6)

$$\dot{\mu} - \beta \mu = -\frac{\partial H}{\partial K} = -\lambda(1 - \tau^K)r + \delta \mu$$

(7)

Transversality conditions are:

$$\lim_{t \to \infty} e^{-\beta t} \lambda B = 0, \text{ and } \lim_{t \to \infty} e^{-\beta t} \mu K = 0$$

By solving these equations, we can yield:

$$U_z(C,1 - L) = U_1(C,1 - L)(1 - \tau^L)w$$

(8)
Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium

\[ \beta - R = \frac{\dot{K}}{\lambda} = \ln \dot{U}_1(C,1-L) \]  \hspace{2cm} (9)

\[ \beta \frac{1-\tau^K}{1-\tau'} r + \delta = \frac{\dot{\mu}}{\mu} = \ln \dot{U}_1(C,1-L) + \ln(1-\tau') \]  \hspace{2cm} (10)

So, erasing \( \beta \) from the system, I get:

\[ R = \frac{1-\tau^K}{1-\tau'} r - \delta + \ln(1-\tau') \]  \hspace{2cm} (11)

**Proposition 1:** If a steady-state Ramsey allocation exists, the optimal capital income tax rate is equal to the investment tax credit rate in the steady state.

Proof: Government’s policy choice is also constrained by the aggregate resource constraint (1), government budget constraint, and the household’s first-order conditions (8)-(9). But it is a little perplexed to write down the Hamiltonian function of the problem, so my proof is based on discrete-time optimal choice. In the discrete-time model, these constraints become (Here I omit the time subscript \( t \))

\[ C + G + K(t+1) - K + \delta K = F(K,AL) \]  \hspace{2cm} (1a)

\[ G + (R+1)B = \tau^K K - \tau' K(t+1) + \tau' K - \tau' K + \tau' wL + B(t+1) \]  \hspace{2cm} (2a)

\[ U_1(C,1-L) = U_1(C,1-L)(1-\tau') w \]  \hspace{2cm} (8a)

\[ \frac{\beta U_1(t+1)}{U_1} \left[ \frac{1-\tau^K (t+1) r(t+1)}{1-\tau'} - \frac{1-\tau' (t+1)}{1-\tau'} (1-\delta) \right] = 1 \]  \hspace{2cm} (9a)

So, the Ramsey problem in Lagrangian form becomes

\[ L = \sum_{t=0}^{\infty} \left[ U(C,1-L) + \psi(-G-(R+1)B + \tau^K K - \tau' K(t+1) + \tau' K - \tau' K + \tau' wL + B(t+1)) + \theta(F(K,AL) - C - G - K(t+1) + K - \delta K) + \mu(U_1(C,1-L)) \right] \]

\[ -U_1(C,1-L)(1-\tau') w + \zeta(U_1 - \beta U_1(t+1) \left[ \frac{1-\tau^K (t+1) r(t+1)}{1-\tau'} - \frac{1-\tau' (t+1) + 1-\tau' (t+1) \delta) \right] \]

Note that the household’s budget constraint is not explicitly included because it is redundant when government satisfies its budget constraint and resource constraint holds. The first-order condition with respect to \( K(t+1) \) is

\[ \theta + \psi \tau' = \beta [\psi (t+1)(\tau^K (t+1) r(t+1) + \tau' (t+1) (1-\delta)) + \theta (t+1)(r(t+1) + 1-\delta) + \theta (t+1)(r(t+1) + 1-\delta)] \]  \hspace{2cm} (10a)

The equation has a straightforward interpretation. A marginal increment of capital investment in period \( t+1 \) increases the quantity of available goods at time \( t+1 \) by the amount \( \lambda (t+1) + (1-\delta) \),
which has a social marginal value $\theta(t+1)$. In addition, there is an increase in tax revenues which is equal to $\tau^e(t+1)r(t+1)+\tau^e(t+1)(1-\delta)$, which enables government to reduce its debt or other taxes by the same amount. The reduction of the “excess burden” equals $\psi(t+1)(\tau^e(t+1)r(t+1)+\tau^e(t+1)(1-\delta))$. The sum of these two effects in period $t+1$ is discounted by the discount factor $\beta$ and set equal to the social marginal value of the initial investment good in period $t$ plus the social marginal value of investment tax credit, which is given by $\theta+s\tau$.

Suppose that government expenditures stay constant after sometime $T$, and the solution to the Ramsey problem converges to a steady state; that is, all endogenous variables remain constant. The steady state version of equation (10a) is

$$\theta + \psi \tau' = \beta[\psi(\tau^K r + \tau'(1-\delta)) + \theta(r+1-\delta)]$$ (10b)

Now with a constant consumption stream, the steady-state version of the household’s optimality condition for the choice of capital in equation (9a) is

$$\beta[(1-\tau^K)r + (1-\tau')(1-\delta)] = 1 - \tau'$$ (9b)

A substitution of equation (9b) into equation (10b) yields

$$(\theta + \psi)\tau(\tau' - \tau^K) = 0$$

Since the marginal social value of goods $\theta$ is strictly positive and the marginal social value of reducing government debt or taxes $\psi$ is nonnegative, and the rental rate is also positive, it follows $\tau = \tau^K = \tau$, which is constant in the steady state.

**Proposition 2:** The capital income tax combined with an investment tax credit at the same tax rate, say $\tau = \tau^K = \tau$, together with zero labor income tax, is non-distortionary, even it is not at the steady state.

**Proof:** Noticing when $\tau = \tau^K = \tau$ and $\tau = 0$, equation (8)-(10) become:

$$U_i(C,1-L) = U_i(C,1-L) w$$ (8c)

$$\beta - R = \ln \dot{U}_i(C,1-L) = \frac{d\ln U_i(C,1-L)}{dt}$$ (9c)

$$\beta - r + \delta = \ln \dot{U}_i(C,1-L)$$ (10c)

So, this tax policy has no effect on capital accumulation. It also has an intuitive explanation: when government levies a lump-sum tax, the optimal household first-order condition becomes the same with this case, thus non-distortionary.
3. Extensions and Applications

Since the capital income tax combined with an investment tax credit is non-distortionary, now I analyze the amount of revenue that can be collected with this tax. Similar with Abel (2007), I begin with calculating total capital income tax revenue which can be collected at time t. Next, to shed light on the size of capital income tax revenue, I focus on the balanced growth path. It is divided into two parts: one for steady state and another for balanced growth path with specific utility function.

3.1 Capital Income Tax Revenue

To calculate taxable capital income in the presence of a constant tax rate, from equation (11), I can obtain

\[ R = f'(K, AL) - \delta \]  \hspace{1cm} (12)

Define \( \gamma = \dot{K}/K \) to be gross growth rate of capital stock at time t. Substitute \( \gamma = \dot{K}/K \) in \( \dot{K}(t) = I(t) - \delta K(t) \) and rearranging yields

\[ I(t) = (\gamma + \delta)K(t) \]  \hspace{1cm} (13)

Define \( TA \) to be taxable capital income\(^4\), so

\[ TA = Kf(K, AL) - I = (R - \gamma)K \]  \hspace{1cm} (14)

If investment, \( I(t) \), exceeds gross capital income, then the taxable capital income is negative, and the owners of capital receive a payment from government. Equivalently, if taxable capital income is negative, the owners of capital are able to receive a tax credit which can be used to pay future capital income tax liabilities. Reduction in income tax liability is granted by government to firms making new investments in certain asset categories, primarily equipment. The investment credit, designed to stimulate the economy by encouraging capital expenditure, has been a feature of tax legislation on and off, and in various percentage amounts. In the long run, however, the dynamic efficiency of the aggregate economy implies that gross capital income exceeds investment, so taxable capital income is positive at steady state.

**Proposition 3:** If the after-tax gross rate of return on bonds is higher than the capital stock growth rate, government can collect substantial positive tax revenue.

\(^4\) Since the investment tax credit rate is equal to the capital income tax at optimal condition, so here I define the taxable capital income is the capital income minus investment.
Proof: To express taxable capital income as a share of output, \( Y \), define \( \eta = \frac{K^T(K, AL)}{Y} \) as the capital share of income at time \( t \), and rearranging to obtain
\[
K = \frac{KF_t(K, AL)}{F_t(K, AL)} = \frac{\eta Y}{R + \delta}
\]
(15)

Then I can get:
\[
T_A = (R - \gamma)K = \frac{R - \gamma}{R + \delta} \eta Y
\]
(16)

Equation (16) shows the value of taxable capital income at time \( t \). So, when \( R > \gamma \), \( T_A \) is positive that government can collect substantial positive tax revenue.

**Proposition 4:** The present discount value of taxable capital income from time \( t \) onward is equal to \( R(t)K(t) \).

Proof: Let \( \mathcal{A}(t) \) be the present discount value of taxable capital income from time \( t \) onward, then
\[
\mathcal{A}(t) = \int_t^\infty \frac{R(t) \exp(-\int_t^s R(x)dx)TA(s)ds}{(R(s) - \gamma(s))K(s)}ds
\]
And we have \( \mathcal{K}(s) = \mathcal{A}(t) \exp(\int_t^s \gamma(x)dx) \), so finally we can yield
\[
\mathcal{A}(t) = \int_t^\infty R(t) \exp(-\int_t^s R(x)dx)(R(s) - \gamma(s))K(s)ds
\]
\[
= \int_t^\infty R(t) \exp(-\int_t^s R(x)dx)(R(s) - \gamma(s))K(s) \exp(\int_t^s \gamma(x)dx)ds
\]
\[
= R(t)K(t) \int_t^\infty \exp(\int_t^s \gamma(x) - R(x)dx)(R(s) - \gamma(s))ds
\]
\[
= -R(t)K(t) \exp(\int_t^\infty \gamma(x) - R(x)dx) \bigg|_{t=\infty}^{s=\infty} = R(t)K(t)
\]
But one necessary and sufficient condition to obtain this equation is \( \exp(\int_t^\infty (\gamma(x) - R(x))dx) = 0 \); therefore, if the after-tax gross rate of return on bonds is higher than the capital stock growth rate\(^5\), the present discount value of taxable capital income at time \( t \) is equal to the after-tax gross rate of return on bonds multiplying the capital stock. Moreover, the present value of capital income tax revenue is \( \tau R K \). In terms of its effects on tax revenue, this policy is equivalent to a one-time government seizure of a fraction \( \tau R \) of the capital stock at time \( t \). Besides, in an economy at the early stage of development, because of low capital stock, government can only collect small, limited amount of revenue. But on the other hand, equation (12)

---

\(^5\) Note that it is only a sufficient condition, not necessary.
Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium

shows that with low capital stock, the marginal physic product of capital can be high thus the R can be high.

Now assume economy takes Cobb-Douglas production function, $Y = K^a (AL)^{1-a}$, then I can calculate the present discount value of taxable capital income from time $t$ onward,

$$X(t) = R(t)K(t) = \alpha F - \delta K$$ (17)

This equation means that the present discount value of taxable capital income from time $t$ onward equals the capital income minus the capital depreciation. In other words, the net capital income is all collected by government. Differentiate equation (17) I can get the first order condition of capital stock that minimize the present discount value of taxable capital income:

$$\alpha^2 K^{a-1} (AL)^{1-a} - \delta = 0.$$ Then solve out when $K = AL(\alpha^2 / \delta)^{1/(1-a)}$, government can collect the minimal present discount value of taxable capital income.

3.2. Steady State

Once economy gets into its stationary state, capital stock and all the relevant macro economic variables, such as government purchase, index of labor-augmenting technical progress, consumption, labor supply, capital income tax rate, etc, all remain unchanged. So, the investment equals capital depreciation, $I(t) = \delta K$, and $\gamma = 0$. In this case, equation (11) shows

$$R' = \beta$$ (18)

Substitute equation (18) into (14), and $\gamma = 0$, we can get

$$T \dot{A} = R' K'$$ (19)

Besides, from equation (15) implies

$$K' = \frac{\eta Y'}{\beta + \delta}$$ (20)

So, equation (19) and (20) can yield

$$\frac{T \dot{A}}{Y'} = \frac{\beta}{\beta + \delta} \eta$$ (21)

Equation (21) shows that the taxable capital income share in total income is $\beta \eta / (\beta + \delta)$ in the steady state. Suppose the rate of time preference is 0.02, the depreciation rate is 0.10, the capital income share of total income is 0.3. In this case, taxable capital income is 0.05 of total income.
3.3. Balanced Growth Path: A Specific Case

For studying a growing economy, models that converge to steady states are not useful, and the appropriate analogue to a steady state is a balanced growth path, defined in this case as an allocation in which consumption, government spending, and both kinds of capital grow at the rate $\gamma \geq 0$ of technology progress, and the labor supply, $L$, is constant. To ensure that such a balanced growth path exists, it is necessary to assume that current time utility function $U$ has the constant elasticity form:

$$U(C, 1 - L) = \frac{[C\varphi(1 - L)]^{-\sigma} - 1}{1 - \sigma}$$

where the coefficient of risk aversion $\sigma$ is positive. When $\sigma$ is equal to one, the utility function becomes in the form of $\log C + \log \varphi(1 - L)$ . When $U$ takes the form of (22), then with $L$ constant, as on the balanced path, the growth rate of marginal utility is just the product of $\sigma$ and the growth rate $\gamma$ of consumption. And function $\varphi$ possesses constant elasticity: $\varphi(1 - L) = (1 - L)^\alpha$, so that along a balanced growth path the present.

Table 1  Taxable Capital Income Share of Total Income

<table>
<thead>
<tr>
<th>Risk aversion $\sigma$</th>
<th>0</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5</td>
<td>2.31</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>4.29</td>
<td>3.75</td>
<td>3.33</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>7.5</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>12</td>
<td>15</td>
<td>16.67</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>15</td>
<td>18.33</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>17.14</td>
<td>20.45</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>18</td>
<td>21.92</td>
<td>23.33</td>
</tr>
</tbody>
</table>

Rate of time preference: $\beta=0.02$
Depreciation rate: $\delta=0.1$
Capital income share of total income: $\eta=0.3$

Value of the utility stream, $\int_0^\infty e^{-\beta t} U(C, M) dt$, is finite.

Using this specific utility function, and the conditions along the balanced growth path, the equations (8)-(10) can be written as:

$$\alpha C = (1 - L) w$$

(23)

$$\beta - \delta = -\sigma \gamma$$

(24)

$$\beta - r + \delta = -\sigma \gamma$$

(25)
Optimal Capital Income Taxation with Investment Tax Credit in General Equilibrium

So, on the balanced path, \( R^* = \beta + \gamma \), then equation (16) becomes

\[
TA = \frac{\beta + \gamma - \gamma}{\beta + \gamma + \delta} \eta Y \quad \text{thus} \quad \frac{TA}{Y} = \frac{\beta + \gamma - \gamma}{\beta + \gamma + \delta} \eta
\]

(26)

Equation (26) shows the taxable capital income share of total income. Especially, when \( \gamma = 0 \), it becomes identical to equation (21). Note that for a positive capital tax revenue, we need the sufficient condition that \( R > \gamma \), which implies \( \beta > (1-\sigma) \gamma \). Besides, by differentiating\(^6\), we can get the capital income tax share increases with increasing \( \gamma > 0 \) while keep \( \sigma \) constant; and it also increases with increasing \( \sigma > 1 + \beta / \delta \), decreasing with increasing \( \sigma < 1 + \beta / \delta \) while \( \gamma \) remains the same.

Table 1 gives the share of taxable capital income along the balanced path for various values of the risk aversion, \( \sigma \), and the technology progress, \( \gamma \). Assume \( \eta = 0.3 \), which is approximately traditional observation of capital income share of total income. Besides, assume the rate of time preference \( \beta = 0.02 \), and a depreciation rate \( \delta = 0.1 \). From the table, we can find out the taxable capital income is higher than the steady state when risk aversion, \( \sigma \), is higher than 1. And column 3, \( \gamma = 0 \), is the case of steady state I calculate in the last part.

4. Taxation on Initial Allocation

Thus far, we have set \( \tau^0(0) = r(0) = 0 \). Now turn to the issue regarding confiscatory tax rates, and suppose that government is free to choose \( \tau(0) \). Define \( \hat{R}(t) := \int_0^t R(s) ds \), the equation (4) and (5) are equivalent to:

\[
\frac{d}{dt} \left[ e^{-\hat{R}(t)} B(t) \right] = e^{-\hat{R}(t)} \left[ (1 - \tau^e(t)) u(t) L(t) + (1 - \tau^s(t)) r(t) K(t) + \tau^s(t) I(t) - R(t) - C(t) \right]
\]

(27)

\[
\frac{d}{ds} \left[ e^{\delta s} K(s) \right] = e^{\delta s} I(s)
\]

(28)

Integrate (27) from \( t = 0 \) to \( t = \infty \) and impose the no-Ponzi condition:

\[
\lim_{t \to \infty} e^{-\hat{R}(t)} B(t) \geq 0
\]

\(^{6}\) Define \( f(\sigma, \gamma) = \frac{\beta + \gamma - \gamma}{\beta + \gamma + \delta} \eta \), then \( f_\sigma(\sigma, \gamma) = \frac{\beta + (\sigma - 1)\delta}{(\beta + \sigma + \delta)^2} \eta \), which implies when \( \sigma > 1 + \beta / \delta \), \( f_\sigma(\sigma, \gamma) > 0 \); \( \sigma < 1 + \beta / \delta \). \( f_\gamma(\sigma, \gamma) < 0 \). And \( f_\gamma(\sigma, \gamma) = \frac{\gamma(\gamma + \delta)}{(\beta + \sigma + \delta)^2} \eta > 0 \), for all \( \gamma > 0 \).
We can get:
\[ B(0) + \int_{0}^{\infty} e^{-\hat{r}(t)} [(1 - \tau^t(t))u(t)\mathcal{L}(t) + (1 - \tau^K(t))r(t)K(t) - (1 - \tau^t(t))I(t) - C(t)] \, dt \geq 0 \] (29)

Integrate (28) from \( s = 0 \) to \( s = t \):
\[ e^{\hat{r}t}K(t) = K(0) + \int_{0}^{t} e^{\hat{r}s}I(s) \, ds \] implies \( K(t) = e^{\hat{r}t}K(0) + \int_{0}^{t} e^{\hat{r}(t-s)}I(s) \, ds \) (30)

Substitute (30) into (29):
\[ B(0) + \int_{0}^{\infty} e^{-\hat{r}(t)} [(1 - \tau^K(t))r(t)e^{\hat{r}t}K(0)] \, dt \]
\[ + \int_{0}^{\infty} e^{-\hat{r}(t)} \left[ (1 - \tau^K(t))r(t) \int_{s}^{t} e^{\hat{r}(t-s)}I(s) \, ds \right] dt - \int_{0}^{\infty} e^{-\hat{r}(t)} [(1 - \tau^t(t))I(t)] dt \geq 0 \]

Change the order of integration in the second line:
\[ B(0) + \int_{0}^{\infty} e^{-\hat{r}(t)} [(1 - \tau^K(t))r(t)e^{\hat{r}t}K(0)] \, dt \]
\[ + \int_{0}^{\infty} K(s) \int_{s}^{\infty} e^{-\hat{r}(t)} (1 - \tau^K(t))r(t) \, dt \, ds - \int_{0}^{\infty} e^{-\hat{r}(t)} [(1 - \tau^t(t))I(s)] ds \]
\[ + \int_{0}^{\infty} e^{-\hat{r}(t)} \left[ (1 - \tau^t(t))u(t)\mathcal{L}(t) - C(t) \right] dt \geq 0 \]

If the coefficient of \( I(s) \) is strictly positive, the representative consumer can attain unbounded wealth by choosing \( I(s) \rightarrow \infty \); if the coefficient of \( I(s) \) is strictly negative, the representative consumer will choose \( I(s) = 0 \) which cannot be consistent with aggregate data.

Hence, \( \int_{s}^{\infty} e^{-\hat{r}(t)} e^{\hat{r}(t-s)}(1 - \tau^K(t))r(t) \, dt - (1 - \tau^t(s))e^{-\hat{r}(t)} = 0 \)

Or equivalently, \( e^{\hat{r}t} \int_{s}^{\infty} e^{-\hat{r}(t)} e^{\hat{r}(t-s)}(1 - \tau^K(t))r(t) \, dt = (1 - \tau^t(s))e^{-\hat{r}(t)} \) for all \( s \)

Substitute this equation back into the budget, we can get:
\[ B(0) + K(0)(1 - \tau^t(t)) + \int_{0}^{\infty} e^{-\hat{r}(t)} \left[ (1 - \tau^t(t))u(t)\mathcal{L}(t) - C(t) \right] dt \geq 0 \]

Notice that \( \tau(0) \) has the same effect as a lump sum tax on the initial capital stock. An optimizing government will thus choose \( \tau(0) \) sufficiently large to finance all government expenditure for all time. In order for the optimal taxation problem to be interesting and avoid the problem of time consistency, a bound must be imposed on \( \tau(0) \). Similarly, a bound must also be placed on \( \tau^t(0) \) for \( t \) early in the planning horizon. Otherwise, as Chamley (1986) pointed out an extremely large value of \( \tau^t(t) \) for an extremely short time interval is asymptotically equivalent to a
5. Concluding Remarks

This paper shows that the combination of investment tax credit and capital income tax can overturn the zero limiting capital tax result in the similar model considered by Chamley (1986). I have mainly suggested three points. First, with the investment tax credit equals to the capital income tax and zero labor income tax, this tax policy is non-distortionary. Secondly, government can collect a substantial amount of tax revenue under the condition that the after-tax gross rate of return on bonds is higher than the capital stock growth rate. Thirdly, in the long run, the optimal capital income tax maybe non-zero.

Why should government provide such credit? The reason lies in that the capital income tax, combined with investment tax credit, can make the optimal capital income tax non-distortionary. Besides, government can collect substantial revenue with such tax policy; therefore it can be used for government purchase, such as public goods, defense, infrastructure, and so on. Therefore, such investment tax credit combined with capital income tax should be implemented.

Additionally, capital income tax is also often treated as a useful tool for the redistribution of income. Here, I have examined that the capital income tax can be collected in a non-distortionary way so that it may be used for the redistribution. However, in order to make the steady state and balance path exist and interesting, a bound must be imposed on the rate investment tax credit as well as that on capital income tax. The economic explanation of this result lies in that in solving for the optimal tax policy, I impose the private sector first-order condition as constraints on government’s allocation problem. In general, the first-order conditions imply that agents’ consumption and saving decisions depend on the anticipated trajectory of future tax rates. In such an environment, a levy on the initial capital stock is an efficient method of raising revenue because it does not distort private-sector decisions. The standard approach implies that government will set the optimal capital tax to its maximum value, 100%, at the beginning of the time horizon, but promise a zero tax rate in the future. However, we cannot make sure that government would be
tempted in future periods to revenge on its promise and levy a confiscatory tax on capital, so the better way is to limit government on its initial tax rate.

Analysis of the capital income tax forces me to think about a series of difficulty questions and some of them are beyond my present knowledge. So, the purpose of this paper is only to consider the optimal capital income tax while the others, such as the balanced growth path of capital stock, consumption, the taxation rates and so on, will keep me thinking; I will try to work further in these areas when I have full knowledge of basic advanced macroeconomics and mathematics.

References